A survey of the experimental facts about atomic phenomena shows that to obtain a theory to fit these facts is not merely a question of finding the correct laws of force between the elementary particles and then applying ordinary mechanics. There are quite general phenomena which will not fit into ordinary mechanics no matter what laws of force are assumed.

For example, the law of black-body radiation discuss in detail.

Also the law connecting harmonic frequencies in a vibrating system.

We are then forced to the conclusion that the mechanics itself must be altered.

The usual mechanics is so neat and simple that one feels that it is a great pity that it has to be altered, and that any alteration one may make must be artificial and ugly of men.

The necessity for an alteration appears in a more natural light however, if one looks at it from the following point. We want our theory to be used only with observable things. It is usually assumed that one can see all there is to see about a mechanical system and that this will not affect the system, but is this assumption justifiable? When the things we observe are very small, like atoms, might not our observations necessarily disturb them? A.M. is built up on the doctrine that there is a theoretical minimum to the amount of disturbance one causes, no matter how carefully one makes the observation.

This is a theoretical limit to how easily we can handle an atomic system.

If we look at the system, it must scatter some light, and there is a theoretical minimum to the quantity of light that may be scattered, namely the light-quantum.

This disturbance which we have to make when we observe the system introduces an uncertainty into the result of the observation. We cannot talk about determinism in the classical sense, because determinism can only apply to a dynamical system that is left undisturbed. But such a system is not
observable, and is thus not the sort of thing one deals with in science.

In general, the quantum mechanics enables one to calculate only the probability for any particular result from observation of a small system.

The actual equations of the QM are of the usual causal type $\frac{\partial^2 (\text{any variable})}{\partial t^2}$ so that we have as much causality as we can expect, but when we come to the interpretation of these equations to get things we can compare with observation, we must introduce statistical ideas, and can calculate only probabilities, that can be interpreted only with

It may not seem to be satisfactory to have a theory based on probabilities in this way, but actually it is what is wanted to describe the physics. Probabilities are the only things which can be obtained experimentally from a study of these small things, and one does not want the theory to give more than the
After the introduction.
I shall now pass on to give the more O.M. theory of a dynamical system.

Let us start by considering the final form of the quantum theory in a classical manner.
The time in which we might allow it to be taken as many years, the lines of
classical mechanics consists essentially in writing down equations of motion and solving them.

In quantum mechanics, we also have equations of motion to solve, very much like the classical case.

The essential new point in the Q.M. is that the energy, position, and momentum of particles, in general, do not derive from
except for the one difference, the multivaluedness of Q.M. follows exactly the same laws as the classical theory.

The fact that \( xy \neq yx \) means that \( x \) and \( y \) cannot be ordinary numbers.

We therefore require a special interpretation for our equations. This will be considered later.

We do not require to know anything about the integer, in order to write down the algebraic equations and solve them, since the

All we require is nothing to replace the numbers \( xy = yx \). These new assumptions are called

For a particle with constant, denote \( x^2, y^2, \) and \( x^2 + y^2 \) are even terms

In the P.B. of classical mechanics, write \( [x, y] \) Thus \( \frac{\partial x}{\partial t} = [x, y] \) in \( h \to 0 \).

in quantum P.B. and may be used to replace classical P.B. always.

For classical mechanics, with \( \frac{\partial}{\partial x} \) we find \( \frac{\partial}{\partial y} \) to first order in \( h \), we find \( \frac{\partial}{\partial y} = \frac{\partial}{\partial x} \) for mirror images on particular, case of this.

This \( H \) in the P.B. of classical mechanics, write \( [x, y] \) Thus \( \frac{\partial x}{\partial t} = [x, y] \) in \( h \to 0 \).

in quantum P.B. and may be used to replace classical P.B. always.

Classical eqn of motion may be written \( \frac{\partial H}{\partial q} = \frac{\partial}{\partial \dot{q}} \) becomes quantum system \( \frac{\partial}{\partial q} \) in \( \frac{\partial}{\partial \dot{q}} \)

There are quantum analogs of motion, expressed in terms of a quantum Hamiltonian \( H \) in the case of noncommutative products.

When it is not, this can usually take over classical Hamiltonian \( H = \sum (\frac{1}{2} m \dot{q}^2 + V(q)) \) for harmonic oscillators

We are now able to write down analogs of motion and solve them in Q.M. or in classical.

eg., for harmonic oscillator \( \frac{\partial}{\partial q} = \frac{\partial}{\partial \dot{q}} \) gives \( \frac{\partial}{\partial q} = \frac{\partial}{\partial \dot{q}} \) \( \dot{q} = \frac{1}{2} m \dot{q} \) \( \dot{p} = -2 m \dot{q} \) \( q = \frac{1}{2} m \omega^2 \dot{q} \) \( p = -\frac{1}{2} m \omega^2 \dot{q} \) \( \dot{p} = -\frac{1}{2} m \omega^2 \dot{q} \) \( \dot{q} = \frac{1}{2} m \omega^2 \dot{q} \) \( \dot{p} = -\frac{1}{2} m \omega^2 \dot{q} \) \( \dot{q} = \frac{1}{2} m \omega^2 \dot{q} \) \( \dot{p} = -\frac{1}{2} m \omega^2 \dot{q} \)

This is complete integral of analogs of motion.

Many of the eqns are the same as in classical theory.
We cannot now consider how our data are to be interpreted. Interpretation has given in essentially that of quantum mechanics.

The interpretation of Q.M. is essentially statistical. We cannot now consider how our data are to be interpreted.

The theory does not in general give us exact information about a single system. But it gives us averages and probabilities. This is really an advantage since it gives us understandable things. The theory applies to an assembly of identical systems.

It does not strictly apply to a large number of actual systems, but to a large number of hypothetical systems describing one actual system about which our knowledge is incomplete. i.e. a Gibbs ensemble, Josephson's work of thermal equilibrium.

Let us be a d.p. variable and let V denote an ensemble of systems, i.e. a number. Note that two elements may be added i.e. we can have V = U + V, with U + V = U and V = V. The total U for all members of the ensemble (x, U) (with some additional later shown necessary). The average x is (x, U)/N. This is average result if x is obtained for the system a posterior phase of this ensemble.

For two members V and U, we must have (x(V), U + V) = (x, U) = (x, V) + (x, U).

Again for two variables x and x', (x' x, U) = (x', U) = (x, U). Then (x', V) is like a product of x and U satisfying the external laws. of two vectors.

It is not a product of numbers, since from x U = 0 we cannot infer x' U = 0, but is rather like a scalar product.

We have (U V) = (U V) = N.

Suppose V is such that for every member of it x has the value x, then 1 x is a number.

We then may U is free and independent from variable x. In some sense for quantum mechanics, the variable is certain to be fixed in the result x = 1.

Then, of course, (x, U) = x = (x, U).

Further, if f(x) is any function of x, (f(x), U) = f(x) = (f(x), U).

Again, if g is any other variable, we should expect (g x, U) = x (g, U) = (g x, U).

We can sum if these results by x U = x U.

This eqn. can be multiplied by any variable on the L.H.S. and enclosed in brackets, with division of common.

This eqn. is the mathematical statement of the fact that x U is a number, not a function of x.

If x and y are any two variables, then in classical mechanics, it would exist a U from w.r. to both of them.

This is not in general true in Q.T.

If U were free for both x and y, we should have (q(x, U), q(x, U)) = (y(x, U)) = (y, x) = (y, U)

or (y - y, U) = 0 which contradicts quantum mechanics.

In general, we can have x U free for both x and y only when x = y.
In classical mechanics, any \( V \) can be expressed as a sum of \( V' \)'s and of which is fixed w.r.t. \( y' \).

\[
V = V_1 + V_2 + V_3
\]

where \( yU_j = y'y_j \); \( yU_j = y''y_j \); \( yU_j = y'''y_j \); \( \ldots \); \( yU_j \); \( \ldots \)

We cannot expect this to hold in Q.M. since \( (xyU_j) = (x'y_j, y'U_j) = (y_j, U_j) \)

and since \( (y_j, V) = (y_j, yV) \) if \( V \) can be expanded in terms of \( y_j \).

Thus \( yV \) is \( y \) times the nature of an ensemble. \( yV \) is a sum of \( yV \)'s times \( y''V \).

In Q.M., it is not true that every \( V \) can be expanded in terms of pure \( U \)'s, but all the same

\[
yU \text{ is the nature of an ensemble. Thus } yV = \sum y_j'U_j \quad \rightarrow \text{Definition of } yV \text{ as an ensemble}
\]

We now come to fundamental laws in interpretation of Q.M.

\[
\langle x, y, U \rangle = \langle x', y', U' \rangle \quad \forall x, y, U \quad \text{identical}
\]

Thus for ensemble \( yU \), equals \( y \) for ensemble \( u \). Proof \( \langle x, yU \rangle = \sum \langle x, y_j', U_j \rangle = \delta(x', x, y_j', y') \)

This law is observation \( U \) can be expressed in terms of pure \( U \)'s w.r.t. \( y \).

We choose in classical mech. \( U \) as assumed to be held generally in Q.M.

This scheme contains all the general assumptions necessary for the interpretation of Q.M.

Further development, we concern only with putting them in a more convenient form. And we are really unnecessary.

In the usual form of this development, we deal entirely with pure ensemble that are pure w.r.t. some variable or other.

But in general, \( yU \) so \( \delta(x, y_j) = \delta(y_j) \).

For the R.T. form, according, in addition to pure ensemble, we have transition ensemble, denoted by \( U_{y'y''} \).

They have the property that for any \( x \)

\[
\langle x, y_j, U_{y'y''} \rangle = \langle x, y_j', U_j \rangle = \delta(x', x, y_j', y')
\]

But

\[
\langle x, y_j, U_{y'y''} \rangle = \langle y''x, y_j', U_j \rangle = \delta(x', x, y_j', y')
\]

\( U_{y'y''} \) is in some way related to the true value of \( y \), that is not really a matter, together of true pure ensemble \( U_{y'y''} \).

It is assumed that any \( V \) can be expanded in terms of pure \( U \)'s, written \( U_{y'y''} \), together with transition \( U \).

In classical mech. we have \( yV = y(U_{y'y''} + \ldots) = yU_{y'y''} \).

Taking \( x = 1 \), we find

\[
\langle y, U_{y'y''} \rangle = y''(U_{y'y''}) = y''(U_{y'y''})
\]

\( \ldots \) \( (U_{y'y''}) = 0 \) if \( y'y'' \)

and deals for a variable in which the total number of particles is zero.
Example: Non-oscillatory. Energy \( W = (p^2 + q^2) \) negligibly small.

Fine possible values of the energy \( W \), i.e., possible \( W \)'s for a pure ensemble \( W_U = W_U \).

It may appear that the theory has developed is totally inadequate, but it is quite possible, using \( \epsilon > 1 \),

\( W' \) may appear value of \( W \) be \( \geq 0 \), since an observer of \( W' \) must give a \( \pi \) result, as that average \( \pi^2 \) be \( \geq 0 \).

We hence

\[
(p + i q)(p - i q) = p^2 + q^2 \pm (\epsilon - 1)q = 2W - \epsilon
\]

\[
(p + i q)(p - i q) = 2W + \epsilon
\]

\[
(p - i q)(p + i q) = (p - i q)(2W - \epsilon) = (2W + \epsilon)(p - i q) \quad (p - i q)(\epsilon - W) = \epsilon W (p - i q)
\]

Hence \( W' \) is an ensemble zero \( W \) to \( W' \), for which \( W = W' \).

\[
W U' = W' U' \quad (W - \epsilon) U' = (W' - \epsilon) U'
\]

Then if we put \( (p - i q) U' = U'' \quad W U'' = (W - \epsilon) U'' \quad U'' \) is zero w.r.t. \( W \) for \( W = W' - \epsilon \).

Then if \( U' \) is a possible value for \( W \), as in \( W' - \epsilon \), unless \( U'' = 0 \), indefinitely,

if \( U'' = 0 \), we have
\[
0 = (p + i q) U'' = (p + i q)(p - i q) U'' = (2W - \epsilon) U'' = 2(W - \epsilon) U'
\]

and since \( U' \neq 0 \), we must have \( W' = \epsilon \).

Hence \( W' \) must be a possible value for \( W \), unless \( W' = \frac{1}{2} \).

Since possible values for \( W \) are \( \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \), an algebraic series would extend to \( \infty \).

For oscillators with frequency \( \nu \), \( W' = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \).

It may seem that the fact that we have been able to get a result is a fluke.

This method does really, however, come quite a class of problem.

Method is to show that if a value for an variable is possible, then so is another.

It is insufficient in many problems that the possible values form a discrete set, e.g., any "mounta..."