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An Exploratory Study of the Use of a Problem-Posing Approach on Pre-Service Elementary Education Teachers' Mathematical Creativity, Beliefs, and Anxiety

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AN EXPLORATORY STUDY OF THE USE OF A PROBLEM-POSING APPROACH ON PRE-SERVICE ELEMENTARY EDUCATION TEACHERS’ MATHEMATICAL CREATIVITY, BELIEFS, AND ANXIETY

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ABSTRACT

This quantitative study examined the notion of mathematical creativity and its relationship to epistemological beliefs of the nature of mathematics and mathematical anxiety. The participants were assessed in this study using the following instruments: *Creative Ability in Mathematics, Mathematics Belief Questionnaire, the General Assessment Criteria, and the Abbreviated Math Anxiety Scale*. The following questions guided this study: What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity? What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs? What relationship exists between elementary pre-service teacher’s mathematical creativity and their mathematical beliefs? What relationship exists between elementary pre-service teacher’s mathematical creativity and mathematical anxiety? The study employed a counterbalance design, randomizing a class of elementary pre-service teachers into two groups and giving a pre- and post-test to determine if significant differences exist in the participants who are exposed to problem posing, divergent thought and invented strategies, that is, a punctuated, intentional experience to mathematical creativity. This difference was also gauged using repeated measures during the study. Furthermore, beliefs and anxiety were correlated with mathematical creativity employing pre- and post-test measures. The findings of this study suggest that mathematical creativity can be fostered and sustained under certain conditions. Also, mathematical beliefs and anxiety, according to the results, are significantly impacted by intentional experiences with mathematical creativity – alternative algorithms, divergent thought, invented strategies, and problem posing.
In *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, the National Mathematics Advisory Panel (2008) submitted six broad recommendations which demand action to ensure the United States’ “mathematical prowess” (p. 1). Furthermore, the National Mathematics Advisory Panel (2008) stated that action “must be taken to strengthen the American people in this central area of learning” (p. 1). Two of the nation’s strengths are its ability to problem solve creatively and thrive economically. However, to sustain any economic stability in a technological world, the nation must advance in the disciplines of science, technology, engineering, and mathematics. If ignored, these recommendations suggest that the United States’ prowess in mathematics is in jeopardy.

With the National Mathematics Advisory Panel’s concerns about the nation’s mathematical prowess, this only heightens the mathematics education community’s awareness of the need of mathematical problem solving. However, the kind of mathematical problem solving must be creative. Why? Because ninety percent of the problems that today’s kindergartners will face when they are in the workplace have not yet been defined or identified. This means the next generation will have to create solutions to problems that do not already exist. More mathematics might not be the exact answer to this dilemma, but rather more of a certain kind of mathematics might answer this question. That is to say, old ways will not necessarily produce new remedies for the next generation’s unseen predicament. This calls for a revision of problems and solution, so that old paradigms are replaced with fresh frameworks to unravel these unknown problems. Therefore, problem solving mathematically must be reconceived to meet the challenges of the future.

This is exactly what the National Council of Teachers of Mathematics [NCTM] (2000) has proposed in its vision for the state of school mathematics:
Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it (p. 3).

NCTM’s vision is the kind of creative mathematics that will produce the changes necessary to meet the demands of the unknown problems our children will face.

How will our nation produce mathematically creative problem solvers in classrooms as described by NCTM’s vision? How can one sustain a nation’s mathematical prowess? One way is to examine the base and foundation of the nation’s educational building blocks to maintain its mathematical might. But who are the key catalysts to creative mathematical classrooms? Are they not the classroom teachers of mathematics? These teachers must not be limited to the secondary mathematics setting, but they must include classroom teachers of elementary mathematics, as well.

Furthermore, to achieve these mathematically creative classrooms, one might look at the pre-service teachers that will shape the students of tomorrow. To shape these students during the crucial elementary years, elementary pre-service teachers are the catalyst to initiate change. The dilemma at hand, according to the literature, is
twofold. First, elementary pre-service teachers have formal or fixed beliefs of mathematics (Collier, 1972; Seaman 2005) which is, in essence, incongruent with NCTM’s vision for the mathematics classroom. Second, is that pre-service teachers experience mathematical anxiety (Battista, 1986; Bursal & Paznokas, 2006; Gresham, 2007; Harper & Daane, 1998; Vinson, 2001).

With little dispute, teachers matter and make a difference in students’ educational experiences. Teachers are the ones who allow students the experiences to create, discover, and explore mathematical relationships in the classroom. These same teachers influence and inspire young learners to new wonders and curiosities about the mathematical universe.

However, with all of the teachers’ potential assets, they can also be viewed as devastating liabilities. For instance, teachers with mathematics anxiety are likely to produce students with mathematics anxiety (Vinson, 2001). Schofield (1981) makes the case that attitudes and beliefs of teachers are directly connected to students’ attitudes and beliefs towards mathematics. When discussing attitudes and beliefs toward mathematics, Vinson (2001) notes that these attitudes and beliefs influence how often mathematics is used, the willingness to pursue advanced work in mathematics, and even the choice of prospective occupations” (p. 90). Elementary pre-service teachers are reported to have high mathematics anxiety with its roots being entrenched in formal or traditional instructional practices (Harper & Daane, 1998). According to their research, Swars, Daane, and Giesen (2006) suggest that mathematics anxiety has a negative relationship with a pre-service teacher’s belief in his or her skills and abilities to be an effective mathematics teacher” (p. 312).

Aiken (1973) claims that teachers directly impact students’ creativity. To problem solve creatively in mathematics this would assume that the teacher poses mathematical problems, asks higher order and reflective questions, encourages groups and whole class discussion, and provides opportunities to observe and explore mathematical relationships (Aiken, 1973). However, to adopt this position of teaching mathematics, according to Collier (1972), one must believe that mathematics is not formal or static but informal and dynamic. Collier (1972) argues, Traditionally, mathematics programs at the elementary school level have emphasized the formal content of mathematics, often
at the expense of helping children see the creative and investigative nature of mathematics‖ (p. 155). In short, although teachers (especially elementary pre-service teachers) possess great potential to impact young mathematical students, they also bare the prospective liability to endanger the nation’s prowess in mathematical creativity.

**Problem Statement**

Several instruments have been developed to test for mathematical creativity. However, the scoring of these tests is time consuming. If other measures or indicators (more efficient and less time consuming) were used to predict mathematical creativity, then mathematical talent could be identified and developed. Not only that, but beliefs are critical and crucial to teachers and students alike. Beliefs are the agents or catalysts from which we (students, teachers, and all people for that matter) act, react, and behave. Therefore, if beliefs about mathematics predict mathematical creativity, then the study of beliefs and mathematical creativity precipitates researchers’ attention. The factors considered in this study are mathematical creativity, mathematical beliefs, and mathematics anxiety.

**Purpose of Study**

The goal of this study was to explore how a punctuated, intentional experience to mathematical creativity and problem posing influences mathematical anxiety, mathematical beliefs, and mathematical creativity. In addition, another goal was to examine how these three variables might relate. That is to say, do these variables (mathematical creativity, beliefs, and anxiety) predict one another? First, if a punctuated, intentional experience to mathematical creativity fosters more mathematical creativity and influences mathematical beliefs and anxiety, this information would be valuable to curriculum designers, mathematics educators, and teachers of mathematics. Second, if relationships exist among these variables, it would inform teachers and teacher educators what beliefs correspond to mathematical anxiety or mathematical creativity (or that mathematical creativity is associated with certain beliefs). Thus, those beliefs could be portrayed to their students in hope to support mathematical creativity and the nation’s mathematical prowess.
Nature of the Study

In the document, *How People Learn: Bridging Research and Practice*, the National Research Council (NRC) (1999) highlighted the following three findings.

1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom (p. 10).

2. To develop competence in an area of inquiry, students must: (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application (p. 12).

3. A ―metacognitive‖ approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them (p. 13).

These findings described, in short, how children and adults learn. First, students are not blank slates; second, both conceptual and procedural understanding are necessary in problem solving; and finally, metacognition must be integral in the learning process. Generally, the NRC illustrates how learning transpires, but does not address specific content domains. To attend to this matter, the Curriculum Planning and Development Division (CPDD) (2006) supplies a mathematics framework. Foremost, ―mathematical problem solving is central to mathematics learning‖ (p. 5). With mathematical problem solving as the centrifuge, five sediments are produced as a result of the clearing factor. These five categories surround, support, and show the essence of mathematical problem solving as a framework. So the mathematical problem solving framework can then be understood in these five classes: concepts, skills, processes, attitudes, and metacognition. CPDD (2006) defines each in the following manner:

- **Mathematical concepts** cover numerical, algebraic, geometrical, statistical, probabilistic, and analytical concepts.
- **Mathematical skills** include procedural skills for numerical calculation, algebraic manipulation, spatial visualization, data analysis, measurement, use of mathematical tools, and estimation.
• **Mathematical processes** refer to the knowledge skills (or process skills) involved in the process of acquiring and applying mathematical knowledge. This includes reasoning, communication and connections, thinking skills and heuristics, and application and modeling.

• **Attitudes** refer to the affective aspects of mathematics learning such as:
  - Beliefs about mathematics and its usefulness
  - Interest and enjoyment in learning mathematics
  - Appreciation of the beauty and power of mathematics
  - Confidence in using mathematics
  - Perseverance in solving a problem

• **Metacognition**, or "thinking about thinking", refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning (p. 3-5).


Couching the CPDD’s mathematical framework with the NCR’s findings of how people learn provided a foundation and lens to view and analyze data collected in this study. As a general thought, the cognitive perspective was used from the NCR
framework. Moreover, the mathematical paradigm for problem solving was employed from CPDD. In particular, attitudes and processes were used to understand mathematical beliefs and anxieties along with mathematical creativity.

![Diagram showing the relationship between Mathematical Anxiety, Mathematical Creativity, Mathematical Beliefs, and Problem Posing.]

Figure 2. The Conceptual Frame.

This quantitative study examined the notion of mathematical creativity and its relationship to epistemological beliefs of the nature of mathematics. The participants were assessed in this study using the following instruments: Creative Ability in Mathematics, Mathematics Belief Questionnaire, and the General Assessment Criteria. The guiding questions that were investigated in this study are the following:

1. What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity?
2. What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs?
3. What relationship exists between elementary pre-service teacher’s mathematical creativity and their mathematical beliefs?
4. What relationship exists between elementary pre-service teacher's mathematical creativity and mathematical anxiety?

The study employed a counterbalance design, randomizing the class into two groups and giving a pre- and post-test to determine if significant differences existed in the participants who were exposed to problem posing, divergent thought and invented
strategies. Differences were gauged using repeated measures during the experiment. Furthermore, beliefs and anxiety were correlated with mathematical creativity employing pre- and post-test measures.

**Operational Definitions**

**Definition of Mathematical Creativity**

Even though no single definition encompasses the essence of mathematical creativity (Haylock, 1987a; Pehkonen, 1997), it is generally described in terms of three major components: fluency, flexibility, and originality. **Fluency** is the frequency or number of responses. **Flexibility** is the shift in categories or methods in the responses to a mathematical task. **Originality** is when a response is novel compared to other responses. Categorically, these responses are “other” (utterly different), and truly represent thought that is “outside the box.”

**Definition of Mathematical Beliefs**

Those attitudes, assumptions, and dispositions about what an entity is, how a particular entity functions, and why it operates in certain ways, is considered to be one’s belief (Philipp, 2007). With that as a foundation, mathematical beliefs would answer questions that pertain to the following questions (although not limited to these particular questions): what is mathematics?, how is it learned?, and how should it be taught? Situating mathematical beliefs on a continuum, one could believe that mathematics is a fixed entity, or a fluid creature.

**Definition of Mathematical Anxiety**

Anxiety is the emotion causing angst, and is often associated with apprehension, concern, fretfulness, nervousness, uneasiness, or worry (Hopko, 2003; Ma, 1999). In the same vein, mathematical anxiety can be seen as those same emotions associated with timed tests, pop quizzes, and word problems in the mathematics environment.

**Assumptions, Limitations, and Scope of Study**

The study aimed to examine mathematical creativity and its relationship to beliefs about the nature of mathematics and mathematical anxiety. It also desired to
understand the effect that a punctuated, intentional experience to mathematical creativity has on mathematical creativity, beliefs, and anxiety. The underlying notion is that individuals have the potential for creativity in mathematics. Guilford (1958) stated, ―Creativity is shared to some degree by all humanity‖ (p. 6) and other researchers like Nickerson, (1999) hold to this view and assumption as well, stating, ―Creative potential is a relatively common endowment‖ (p. 407). Therefore, the assumption of this study was that all participants have the potential to manifest mathematical creativity. It was also assumed that, if a relationship does exist, then this relationship would inform teachers and teacher educators what beliefs correspond to mathematical creativity. Furthermore, the educational community in mathematics (curriculum designers, educators, and teachers) might benefit from this information in hopes to support mathematical creativity and the nation’s mathematical prowess.

In addition, the researcher must disclose his personal propensity and pedagogical position when it comes to defining what mathematics is and how it is learned. First, although mathematics appears to be a depository of fixed knowledge, it can be seen from the perspective of a fluid, ever growing corpus that can be understood from differing vantage points. Which is to say, mathematics is more than an arbitrary and static set of rules; it is an activity performed. This consists of building, conjecturing, counting, drawing, describing, developing, discovering, explaining, exploring, formulating, investigating, justifying, predicting, representing, verifying, and the like. In a word, mathematics is both a noun and, more importantly, it is also a verb. Static, it is, to those who passively observe the discipline, but to those who actively study these relationships, mathematics is quite dynamic.

Second, mathematical learning does not consist of passive learners that absorb skills and concepts from a disseminating instructor. Students must be engaged, and students must be participating in the learning and understanding of mathematics. Therefore the learning of mathematics is viewed as an active process. In this process, students construct knowledge from previous knowledge and from new, and meaningful, experiences. Then these mathematical experiences are internalized and synthesized. It is through these reflective practices that true mathematical understanding begins.
In summary, this is the researcher’s perspective: mathematics can be viewed as a body of flux (ever growing and changing) and the learning of mathematics includes active participants personally constructing knowledge and making sense of mathematical situations. Therefore, mathematics and the learning of it possess the real potential to manifest a creative quality. The dynamic nature of mathematics and the active involvement in learning it both supply and sustain a culture that supports mathematics as a creative endeavor.
The central theme to this study and chapter are constructed around mathematical creativity. First, the notion of creativity, in general, will be discussed. Then, more importantly, mathematical creativity will be examined. This includes definitions, instruments, designs, populations, and results of mathematical creativity. Finally, this chapter will consider beliefs and anxiety, as they relate to mathematics.

The variables in the study are mathematical creativity, mathematical beliefs, and mathematical anxiety. It is hypothesized that mathematical creativity is predictive by mathematical beliefs and mathematical anxiety.

**Defining Creativity**

No monolithic definition exists for creativity. Nevertheless, researchers have defined creativity in manifold ways. However, certain themes do appear in defining what is creativity.

**Process and Product**

One way that researchers have examined creativity is to observe the process or the product (Aiken, 1973; Guilford, 1964). Guilford (1964) uses the terms —creative potential" and —creative production" to differentiate between mental processes and tangential products. Using differing approaches of thinking about an idea or concept, new analysis of situations or problems to understand it better, or amalgamation of several thought processes may be considered a creative process (i.e., combining or creating analogies or metaphors to attack particular dilemmas) (Nickerson, 1999).

**Individual and Societal**

Creativity can be dissected or bifurcated into individual and societal creativity. Individual creativity would pertain to problem solving in different areas at the personal
level – at work, home, school, etc. Whereas, societal creativity creates new products for the masses – scientific findings, movement in art, inventions, and social programs (Sternberg & Lubart, 1999). Nickerson (1999) describes individual creativity as the rediscovery of the unknown concepts and relationships to the individual despite the fact that those concepts or relationships are known to the rest of the world.

Originality and Utility

Mayer (1999) notes in his amalgamation or synthesis that creativity contains originality (or novelty) and usefulness as two defining characteristics. For instance, Nickerson (1999) states, “Although not everyone considers it possible to articulate clear objective criteria for identifying creative products, novelty is often cited as one of their distinctive characteristics, and some form of utility – usefulness, appropriateness, or social value – as another” (p. 392). Likewise, “Creativity is the ability,” according to Sternberg and Lubart (1999), “to produce work that is both novel (i.e., original, unexpected) and appropriate (i.e., useful, adaptive concerning task constraints)” (p. 3). From a cultural point of view, Lubart (1999) states, “Creativity from a Western perspective can be defined as the ability to produce work that is novel and appropriate…Novel work is original, not predicted, and distinct from previous work” (p. 339). In response to their own question, “What do we mean by creative work?” Gruber and Wallace (1999) answer: “Like most definitions of creativity, ours includes novelty and value: The creative product must be new and must be given value according to some external criteria. But we add a third criterion, purpose – creative products are the results of purposeful behavior – and a fourth, duration – creative people take on hard projects lasting a long time” (p. 94). Martindale (1999) crafts the definition with the following explanation, “A creative idea is one that is both original and appropriate for the situation in which it occurs. It would seem that creative productions always consist of novel combinations of preexisting mental elements” (p. 137). As noted earlier, in each one of these cases, uniqueness and usefulness are considered to be the certain tenants of what it means to be creative.

Originality and utility in creativity are like counterbalancing weights. Too much of one over the other will disrupt the creative balance. For instance, Amabile and Tighe
(1993) point out that the —product or response cannot merely be different for the sake of difference; it must also be appropriate, correct, useful, valuable, or expressive of meaning” (p. 9). In other words, originality for its own sake is deemed void without its utility.

Note the order, novelty precedes utility. The product or process of creativity must be new or innovative. This goes without saying, but one cannot simply regurgitate utility or usefulness in and of itself. The product must go beyond this and also have the element of originality. Some have qualified their definition of creativity by stressing the importance that —creativity is reflected in the generation of novel, socially valued products” (Mumford, Reiter-Palmon, & Redmond, 1994, p. 3). —Creative products, be they poems, scientific theories, paintings or technological advances, are both novel and acknowledge to be valuable or useful in some way” (Gilhooly, 1982, p. 123).

**Radical Novelty and Orthodox Novelty**

Cropley (2006) has noted the regulation of creativity in disciplines. Furthermore, some disciplines are more accepting of the degree of creativity than others. For instance, the arts would tolerate —radical novelty” while the sciences would only allow —orthodox novelty” (Cropley, 2006). To support the orthodox position, Nickerson (1999) states, —For a activity or product to be perceived as creative in science, it must be novel, but not too great a departure from prevailing ideas; if it does not connect with existing theory, it is likely to be ignored” (p. 393). Ward (1995) echoes these same notions with this statement, —Creative thinking moves beyond what has been done only slowly, and when it does, it is more as a modification of the past than rejection of it” (p. 71). At any rate, some discipline are more permissive than other when it comes to innovation, and Lubart (1999) would state that same is true also of culture, too.

**Creativity and Problem Solving**

Some have suggested that problem solving is essentially creative in nature. For instance, Gardner (1989) states, —In my view, creativity is best described as the human capacity regularly to solve problems or fashion products in a domain, in a way that is initially novel but ultimately acceptable in a culture” (p.14).
Fostering Creativity

Many have suggested ways or manners to develop creativity. Nickerson (1999) proposes the following to enhance creativity: establish purpose and intention, build basic skills, encourage acquisition of domain-specific knowledge, stimulate and reward curiosity and exploration, build motivation (especially intrinsically), encourage confidence and willfulness to take risks, focus on mastery and self-competition, promote supportable beliefs about creativity, provide opportunities for choice and discovery, develop metacognitive skills, teach techniques and strategies for facilitating creative performance, and provide balance. For the classroom teacher, Sternberg (1996) offers his “Strategies by the dozen” to make students more creative: (a) serve as a role model for creativity, (b) encourage questioning of assumptions, (c) allow mistakes, (d) encourage sensible risk-taking, (e) design creative assignments and assessments, (f) let students define problems themselves, (h) rewards creative ideas and products, (i) allow time to think creatively, (j) encourage tolerance of ambiguity, (k) point out that creative thinkers invariably face obstacles, (l) be willing to grow, and (m) recognize that creative thinkers need to find nurturing environments. In one of the succeeding sections (Fostering Mathematical Creativity), these particular strategies will be addressed to promote creativity in mathematics.

Defining Mathematical Creativity

Definitions

Universally, no single definition exists for mathematical creativity. Often it is functionally defined and examined (Aiken, 1973; Chamberlin & Moon, 2005; Haylock, 1985; Jensen, 1973; Krutetski, 1976). For instance, Jensen (1973) operationally defined mathematical creativity as, “the ability to give numerous, different and applicable responses when presented with a mathematical situation in written, graphic, or chart form” (p. viii). Similarly, Chamberlin and Moon (2005) state, “Mathematically, creativity may be existent when a nonstandard solution is created to solve a problem that may be solved with a standard algorithm” (p. 38). In other words, mathematical creativity can be seen as invented algorithms and strategies or even alternative approaches to a standard problem. Another way mathematical creativity has been
defined is overcoming fixations and divergent products (Haylock, 1985, 1997). Tammadge, as cited by Haylock (1985), sees the definition as creating new relationships and associations. Russian psychologist Krutetski (1976) states that mathematical creativity can be manifested in five different ways: (a) through problem posing, (b) using alternative methods, (c) by inventing proofs, by generating formulas, and (d) creating unique methods to solve mathematical problems.

Some (Aiken, 1973; Haylock, 1997; Wu & Chiou, 2008) have pointed out that creativity can be bifurcated into a process or a product. That is to say, mathematical creativity is a process of thought, and it is a product manifested in fluency, flexibility, and originality. Sheffield (2000) defines these three products in the following manner: fluency – the number of different correct answers, methods of solutions, or new questions formulated; flexibility – the number of different categories of answers, methods, or questions; originality – solutions, methods, or questions that are unique and show insight.

**Fostering Mathematical Creativity**

In the area of mathematics education, many have suggested several ways to foster creativity. Aiken (1973) asserts, “A creative teacher produces creative students” (p. 420). Following that declaration, he describes the teacher fostering creativity as one who poses problems, asks question, encourages discussion, and provides opportunity to observe and explore —in the mathematical laboratory” (p. 420). In the elementary school, Jensen (1976) indicates that encouragement of students to find multiple methods, alternative algorithms, or unique solutions to problems increases student’s problem solving ability and divergent thinking. Whitcombe (1988) in *Mathematics: Creativity, Imagination, Beauty* calls for the balance of mathematics with an ABC model for mathematics. First, “A” represents the traditional and logical algorithms of mathematics. Then, “B” is the beauty or aesthetic speculation of mathematics. That is, noting the —structure, form, relations, visualization, economy, simplicity, elegance, and order” of mathematics (p. 15). Finally, he states that “C is for the intuitive and creative aspect of mathematics seen in —problem solving, investigations, pattern making, originality, speculation, thinking, concepts, and strategies” (p. 15).
Because problem solving and problem posing are central to the nature of mathematics (and for that matter mathematical thinking) Silver (1997) proposed that mathematical creativity could and should be developed through inquiry-oriented mathematics instruction that employs ill-structured or open-ended problems during the problem solving and posing process. By doing so, Silver maintains, students will enhances "greater representational and strategic fluency and flexibility and more creative approaches to their mathematical activity" (p. 79). If mathematical creativity manifests itself in the classroom, Sriraman (2004) conjectures that, "students should be given the opportunity to tackle non-routine problems with complexity and structure – problems which require not only motivation and persistence but also considerable reflection" (p. 32). In another place, Sriraman (2005) argues for five overarching principles to maximize creativity at the K-12 level:

(a) the Gestalt principle – freedom of time and movement,

(b) the aesthetic principle – appreciating the beauty of unusual solution/connections to the Arts and Sciences,

(c) the free market principle – encouraging risk taking and atypical thinking,

(d) the scholarly principle – view creativity as contributing to, challenging known paradigms and extending the existing body of knowledge, and

(e) the uncertainty principle – open-ended and/or ill posed problems and tolerating ambiguity (p. 27).

When Romey (1970) defines creativity, he contends that it is "the ability to combine ideas, things, techniques, or approaches in new ways" (p. 4). Furthermore, he maintains that this is not only from the perspective of the person doing the actual creating but also from the perspective of the person viewing the creative work. It is at this point he connects the person doing and viewing the creative work as people in the classroom. Therefore, Romey (1970) commends several different ways to model creativity to students, such as creativity in establishing the order in which topics are considered, posing problems, lesson planning, conducting lessons, laboratory activities, questioning strategies, and evaluation.

Zeddies (1981) urges teachers to use student's creative responses and alternative algorithms to assist struggling students who do not understand the
A conventional algorithm. Reed (1957) in *Developing Creative Thinking in Arithmetic* enumerates ten questions for “the good creative teacher” to positively affirm:

1. Do I help my children identify the arithmetical problems significant to them in terms of their experimental background of living?
2. Do I allow time for my children to take action in thinking through the solution of a problem before suggesting methods of solving it?
3. Do I through my questions and directions encourage my children to think for themselves?
4. Do I encourage my children to seek for solutions to a problem instead of the solution?
5. Do I show sincere appreciation and approval for a child’s creative efforts although the answer was unsatisfactory?
6. Do I place as high a value on creative thinking as I do correct responses to drill problems?
7. Do I encourage my children to evaluate their own methods used in solving a problem?
8. Do I recognize the differences among my children in their ability to solve problems creatively?
9. Do I make a conscious effort to help my children understand that arithmetic is a quantitative way of thinking and that it offers opportunities for creative thinking?
10. Do I evidence creativity in my teaching methods, in the way I organize curriculum content, and in my own personal behavior? (p. 12)

In viewing mathematical creativity, Parr (1974) makes the case, using the four stages by the Gestalt, that the phenomenon of mathematical invention has implications for the teaching of mathematics and not just for mathematicians. In the preparation stage, two poles are necessary – knowledge and intuition. Classrooms generally espouse knowledge, but can foster intuition by providing situations for students to make conjectures about mathematical scenarios. Clearly, this requires a supportive classroom atmosphere where mathematical experiments transpire and mathematical guessing is encouraged. During the incubation stage, the mind rests and forgets...
superfluous information, so that a solution path is highlighted. At this stage, Parr (1974) encourages classroom teachers, “to collapse the process into shorter periods of time” (p. 58) by presenting pertinent information for the obvious reason of natural time constraints of school mathematics. This leads to the illumination stage, the actual creative activity, which he points out “is not entirely a logical intellectual activity” (p. 58). Nevertheless, it is the “ah-ha” moment of creation or discovery. During the final stage of elaboration, students should talk about the mathematics – its aesthetics, beauty, and elegances.

The traditional tragedy of school mathematics is the overemphasis on “kill-and-drill” or “theorem-proof, theorem-proof” routine (Parr, 1974; Pehkonen, 1997). Mann (2005) states, “Creativity needs time to develop and thrives on experience” (p. 19). This sentiment corresponds to Silver’s (1997) statement that creativity, “is often associated with long periods of work and reflection rather than rapid, exceptional insight; and is susceptible to instructional and experiential influences” (p. 75). Furthermore, Haylock (1997) expounds that overcoming (both algorithmic and content) fixations to produce divergent products strikes at the heart of mathematical creativity. McGannon (1972) argues that mathematical “mechanization,” “problem solving rigidity,” and “functional fixity” are inhibiting factors that “militate against our [students] attacking new problems with an imaginative approach” (p. 12). In sharp contrast, other have indicated that ill-structured, open-ended, or multiple solution problems along with problem posing support creativity in the mathematical classroom (Becker & Shimada, 1997; Hashimoto, 1997; Haylock, 1997; Kwon et al, 2006; Leikin & Lev, 2007; Pehkonen, 1997; Silver, 1997).

**Problem Posing, Mathematical Curiosity, and Mathematical Creativity**

In the literature there seems to exist an interconnected relationship between problem posing, mathematical curiosity, and mathematical creativity. Some have suggested that problem posing solicits mathematical curiosity, and others imply that mathematical curiosity is a gateway to mathematical creativity, and yet, it is also believed that problem posing fosters mathematical creativity.
Using Brown and Walter’s work, problem posing is defined by Shaw and Aspinwall (1999) with a five step pose-and-probe rubric: first of all, identify a problem; then identify the attitudes of the problem; third, apply “What if not …?” to one or more of the attributes; then, investigate these questions; finally, relate the investigations to the original problem (p. 192). Problem posing to some degree is problem extending. That is to say, problem posing, in some sense, advances the problem to the $n^{th}$ degree (to use the common vernacular or nomenclature) by altering the initial conditions or assumptions of the original problem.

In describing how children learn, Ginsburg and Baron (1993) make the argument that children are “natural learners” and “naturally curious” (p. 5). This statement suggests striking implications for the classroom as it pertains to problem solving. First, that children are not blank slates, but rather they possess some understanding of how their world works (Donovan et al., 1999). That is to say, children are “in the ruff” problem solvers – natural learners. Second, they desire to know how their world works. The first implies the second. If learning is inherent in the learner, so is the desire to learn. This desire, in part, is what we call curiosity.

Knuth (2002), in *Fostering Mathematical Curiosity*, made a similar comment, “Young children have a natural curiosity about the world in which they live” (p. 130). However, the sad reality is that innate curiosity, so it appears, erodes with time and slowly dissipates and disappears. Knuth suggests at least one way to foster mathematical curiosity. Knuth (2002) defines, in part, mathematical curiosity with three particular facets that hinge around the desire to learn mathematics, know mathematics, and explore mathematics. Using Brown and Walter’s (2005) notions of problem posing, Knuth elaborates and demonstrates how curiosity can be fostered using such a technique.

Problem posing then appears to be a platform to solicit mathematical curiosity. NCTM (2000) also validates its role and position in the mathematics curriculum. However, problem posing offers much more than a natural invitation and reason to study mathematical problem solving, English (2003) presents three benefits to problem posing. From English’s perspective, “problem posing is a natural part of our everyday lives” (p. 187). That is to say, just as children are natural learners and naturally curious,
so it is that they would inherently ask why and problem pose. These benefits, according to English (2003), are the following: —problem posing (a) promotes students’ conceptual development, (b) plays a crucial role in students’ understanding of problem structure and design, and (c) enhances students’ access to important mathematics” (p. 188). In short, problem posing not only fosters mathematical curiosity, but it also forges mathematical understanding.

Problem posing is the forgotten component of the mathematics curriculum. In fact, it is the counterpart or complement of problem solving. In problem solving metacognition is a vital component. One of the aspects to metacognition is reflecting on what was done in the problem solving process. In Polya’s (1945) four step approach to problem solving (understand, plan, carry out, look back), he, to some degree, foreshadowed this point with the last step – look back or reexamine. Reflecting, according to Wilson, Fernandez, and Hadaway (1993), has five learning advantages when used after solving a problem. Namely, reflecting possesses the potential for students to develop and explore problem contents, extend problems, extend solutions, extend processes, and develop self-reflection. It is inescapable how the majority of these advantages relate to problem posing and, as noted previously, a particular catalyst to mathematical curiosity is problem posing.

Notice how NCTM (2000) connects problem posing to problem solving, —teachers should regularly ask students to formulate interesting problems based on a wide variety of situations, both within and outside mathematics. … These experiences should engender in students important problem-solving dispositions—an orientation toward problem finding and problem posing; an interest in, and capacity for, explaining and generalizing; and a propensity for reflecting on their work and monitoring their solutions” (p.258). This statement casts a vision for problem solving and posing and hooks an intellectual slate of curiosity to it. Twenty years earlier, in An Agenda for Action (1980), it was stated explicitly, —the mathematics curriculum should be organized around problem solving…Fundamental to the development of problem-solving ability is an open mind, an attitude of curiosity and exploration, the willingness to probe, to try, to make intelligent guesses” (p. 3). In short, NCTM values and emphasizes problem solving, problem posing, and mathematical curiosity.
Some have believed in the power of mathematical curiosity. Many years ago, Polya (1945) wrote, “Your problem may be modest; but if it challenges your curiosity and brings in to play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime” (p. v). That provocative statement is similar to one in more recent times. In defending the problem solving approach, Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, and Wearne (1996) argued that, “students should be allowed and encouraged to problematize what they study, to define problems that elicit their curiosities and sense-making skills” (p. 12). In many regards, to problematize is to problem solve or problem pose. In any case, it incites mathematical curiosity.

Problem posing not only solicits mathematical curiosity, but according to Silver (1997) it also sets the stage for mathematical creativity. He continues to suggest that problem posing is not inherently creative in and of itself (although many disciplines do consider it to be so), but it is the interplay or interaction between problem solving and problem posing that secretes the environment for mathematical creativity (Silver, 1997). In the common vernacular, problem posing is one way to get the creative juices flowing in mathematics.

However, it is not the only way. Silver (1997) also suggests that “inquiry-oriented mathematics instruction” is another avenue to foster mathematical creativity. Inquiry-oriented instruction would be functionally defined as the use of ill-structured or open-ended problems. In an ill-structured problem, according to Mayer and Wittrock (2004), “the given state, goal state, and/or allowable operators are not clearly specified” (p. 288). Shimada (1997), in The Open-Ended Approach: A New Proposal for Teaching Mathematics, describes open-ended problems as having multiple correct answers. In the same volume, Sawada (1997) added to that description by stating, an open-ended problem is where the, “solutions or answers are not necessarily determined in only one way” (p. 23). Problem solving, in short, that employs inquiry-oriented instruction through the implementation of ill-structured or open-ended problems allows students to think...
about a problem from multiple entry points to attack the problem and/or multiple solutions to answer the problem.

These multiple approaches (or methods) to solve the problem and these multiple solutions to the problem are one of the ways to quantify mathematical creativity. Mathematical creativity has no singular definition which is agreed upon (Haylock, 1987a). However, there are several criteria in which it is evaluated. The three key components are fluency, flexibility, and originality (Balka, 1974a; Haylock, 1987a; Mann, 2005, 2006; Silver, 1997). To discover and develop mathematically promising students, Sheffield (2000) suggests seven criteria: depth of understanding, fluency, flexibility, originality, elaboration or elegance, generalizations and reasoning, and extensions.

In conclusion, mathematical curiosity and mathematical problem posing are connected, mathematical problem posing and problem solving are inseparable, and problem posing and problem solving are conduits to mathematical creativity. *An Agenda for Action* (1980) states: ―Problem solving, which is essentially a creative activity, cannot be built exclusively on routines, recipes, and formulas‖ (NCTM, 1980). Therefore, non-routine questions and ill-structured problems must be employed in the problem solving classroom.

**Instruments**

Over the years, several instruments have been developed to measure mathematical creativity (e.g. Balka, 1974a; Foster, 1970; Kim & Cho, 2003; Jensen, 1973; Singh, 1987; Sheffield, 2000). There are a number of common threads shared in each instrument. For one, all are scored by three, if not more, standard notions: fluency, flexibility and originality. Many of the problems are open-ended or open-response. In either case, this allows for variability in the answer, which in return permits the problem to be scored accordingly. Some of the instruments employ problem solving while others use problem posing.

**Scoring Rubric to Encourage Depth and Creativity**

Sheffield (2000), who originally wrote her doctoral dissertation on mathematical creativity, has offered a scoring rubric to discover and assess young mathematicians. This rubric contains seven assessment criteria (*depth of understanding, fluency,
flexibility, originality, elaboration or elegance, generalization and reasoning, and extensions). She defines each in the following manner:

1. Depth of understanding – the extent to which core concepts are explored and developed
2. Fluency – the number of different correct answers, methods of solutions, or new questions formulated
3. Flexibility – the number of different categories of answers, methods, or questions
4. Originality – solutions, methods, or questions that are unique and show insight
5. Elaboration or elegance – quality of expression of thinking, including charts, graphs, drawings, models, equations, and words
6. Generalizations and reasoning – patterns that are noted, hypothesized about, and verified
7. Extensions – related questions to be explored, especially those involving “why?” and “what if?” (p. 419)

Three of those seven are commonly agreed upon to measure mathematical creativity, namely: fluency, flexibility, and originality (Haylock, 1987a; Mann, 2005, 2006; Silver, 1997). Elaboration or elegance, another assessment criterion by Sheffield, is also mentioned in the literature as a measurement of mathematical creativity (Haylock, 1987a).

The strength of this scoring rubric is its power to quantify specific and precise aspects of mathematical problems being solved. It is designed not to understand the student’s process of thought like other instruments. Rather, it was intended to unearth the student’s product of thought on paper for a particular mathematical problem. Problems that are open-ended or ill-constructed would be ideal for Sheffield’s scoring rubric. However, the last two criterions (generalization and reasoning and extensions) would require questions or prompts to solicit those particular kinds of responses.

Earlier criteria have been developed to measure mathematical creativity. Balka’s (1974b) work is most notable in this area. Using two mutually exclusive categories, he devised six different criteria of creative mathematical ability into divergent and
convergent activities. (See Appendix F.) For the divergent criterion, Balka employed fluency, flexibility, and originality as the scoring mechanism.

**General Assessment Criteria of Problem Posing**

Silver (1997) suggests that mathematic problem solving and problem posing can foster mathematical creativity (See Appendix E). Through ill-structured or open-ended problems, according to Silver, students are likely to develop fluency, which is a key component in mathematical creativity. Using Brown and Walter’s (2005) “What-if-not” model of problem posing, Silver argues that students can, “develop greater representational and strategic fluency and flexibility and more creative approaches to their mathematical activity” (p. 79).

In order to assess and understand how students problem pose, Silver and Cai (1996) have formulated a multiple-step coding scheme. Students’ problem posing responses are first coded into three categories: nonmathematical questions, mathematical questions, and statements. Although the primary interest is the mathematical question category, the other two may provide insight into the students’ understanding or values. At any rate, the mathematical questions are then coded at another level. Namely, are the mathematical questions solvable or non-solvable? Finally, the solvable mathematical questions are coded according to semantic and linguistic syntactic analysis.
To make this coding scheme more assessable to researchers and teachers alike, Silver and Cai (2005) have proposed three general assessment criteria: quantity, originality, and complexity. Clearly, quantity, as an assessment criterion to problem posing activities, is exactly what it says – the number of generated responses to a problem prompt. Originality is a relative criterion, but all the same, it is unique or less frequently seen in responses. Complexity is, “the sophistication of the mathematical relationship embedded in the problems” (Silver & Cai, 2005).

The strength of Silver and Cai’s general assessment criteria is its design to examine responses to problem posing situations by students. With its three criterions, the general assessment criteria are broad enough to problem posing situations spanning from the elementary to the post-secondary levels. However, its individual criterions are not as precise at Sheffield’s scoring rubric. In examining mathematical creativity, Silver and Cai’s general assessment criteria provide a promising instrument to evaluate problem posing activities.
Creative Ability in Mathematics Test

To measure mathematical creativity, Balka (1974a) designed an instrument that examines divergent responses to particular situations (See Appendix B). The instrument provides several scenarios and the test subject is to generate as many characteristics, relationships, or questions as possible. These responses then are scored with the three commonly used categories in mathematical creativity: flexibility, fluency, and originality. For each scenario, flexibility was scored for the number of responses. Fluency was scored for different categories of answers. As for originality, its score was weighted. Common answers were weighted with a score of zero. Answers that occur only in 4.99% of the sample population’s response or less to a given situation are considered uncommon and it is weighted with a score of one. A response is weighted with a score of two, if the answer occurs in less than 2% of the sample population’s response to a given situation.

Mathematical problem solving is a vast domain. Mathematical problem solving has some less charted waters in the regions of problem posing and mathematical creativity. The scoring rubric by Sheffield (2000) offers some hope for developing depth and creativity. Problem posing appears to have legitimate assessment criteria (Silver & Cai, 2005). The Creative Ability in Mathematics Test developed by Balka (1974a) still possesses its appeal to measure mathematical creativity.

Professional and K-12 Mathematical Creativity

To understand mathematical creativity, it is helpful to notice two distinctions. For instance, Sriraman (2005) espouses the following difference between professional and K-12 mathematical creativity. Mathematical creativity at the professional level is “(a) the ability to produce original work that significantly extends the body of knowledge, and/or (b) the ability to open avenues of new questions for other mathematicians” (p. 23). However, at the K-12 level, Sriraman (2005) defines mathematical creativity to be “(a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination” (p. 24). Although the two definitions are different, they do have similarities,
namely the second part to both definitions. However, Vivona (1999), desiring a unified theory of mathematical creativity with support from neuroscience and neural networks, does not include K-12 level into his conversation about a theory for mathematical creativity.

Nevertheless, many of the efforts in studying mathematicians have focused on personality characteristics (e.g., Helson & Crutchfield, 1970a, 1970b). Testing Hadamard’s (1945) four stages for mathematical creativity (preparation, incubation, illumination, and verification), Sriraman (2004) uses the qualitative method of analytic induction to verify the stages for five mathematicians. “Trying to better understand the process of creativity,” Sriraman concludes, “the Gestalt model proposed by Hadamard (1945) is still applicable today” (2004, p. 31). Others have suggested using these same stages but for instructional purposes for school mathematics (Parr, 1974; Romey, 1970).

Several studies exist that examine mathematical creativity in the K-12 level. Mann (2005) used a standard multiple linear regression to predict mathematical creativity in eighty-nine seventh graders. This study examined five independent variables: achievement in mathematics, attitude towards mathematics, student perceptions of creative ability, teacher perceptions of mathematical talent and creative ability, and gender. Of the five variables, mathematical achievement was the strongest predictor. Gender, attitude towards mathematics, and self-perceptions of creativity also predicted mathematical creativity. In his concluding remarks, Mann (2005) suggested, “there is relationship between mathematical experiences (knowledge and skills) and creativity in mathematics” (p. 65).

In considering the variables of mathematical creativity, numerical aptitude, and mathematical achievement, Jensen (1973) studied the relationship of these variables with sixth grade students. The results found a low correlation between mathematical creativity and numerical aptitude. Furthermore, mathematical creativity and mathematical achievement prove to be slightly high but still a low correlation.

Haylock (1978) studied 136 students of the age 14-15 years. Using distinct instruments to measure general creativity, mathematics attainment, and mathematical creativity, Haylock concluded that general creativity combined with mathematical skills
and knowledge in students does not equate to mathematical creativity. In other words, mathematical creativity is its own — specific ability" and not a composite construct of mathematics and creativity (p. 25).

In a Korean study with seventh grade students, Kwon, Park, and Park (2006) cultivated divergent thinking (or creativity) in mathematics via an open-ended approach with twenty designed lessons. Using analysis of covariance (ANCOVA), Kwon and colleagues found the gains in the treatment group to be significant as opposed to the comparison group in all three areas of mathematical creativity: fluency, flexibility, and originality.

Using 180 seventh grade students, Banghart and Spraker (1963) investigated the effect, — the role of group influence on creativity during mathematical problem solving" (p. 258). This experiment considered grouping and not grouping, along with ability levels (high, average, and low). The findings suggested no significant difference in mathematical creativity among the individuals or the groups when problem solving. But significant differences were reported among the ability levels.

When exploring the relationship between creativity tests, subject-preference surveys, teacher-rating forms, and achievement tests, Prouse (1967) sampled 312 seventh grade students. Prouse found that divergent thinking correlated higher with creativity than convergent thinking. In addition, he reported that teacher-rating forms correlated with students’ creativity scores. For that particular sample, Prouse found that females scored higher than males on the creativity test. The subject-preference survey also yielded significant results among the subjects.

Eighteen students from the 10th and 11th grade, using clinical interviews, were examined by Leikin and Lev (2007). Using multiple solution tasks, the researchers investigated its utility to examine mathematical creativity. Three different levels (gifted, proficient, and regular) of students (with six in each level) were interviewed individually. The students were given a series of tasks to solve using multiple solutions. The tasks were labeled conventional (textbook problems), partly non-conventional, and non-conventional problems. Leikin and Lev (2007) reported several findings. Gifted students manifested greater novelty than the other two groups of students. Corresponding to that finding, a greater difference in novelty was found between the
proficient and regular group than between the gifted and regular group. In short, the researchers reported that gifted students were more mathematically creative than proficient students who were more mathematically creative than regular students, and they, also, supported the notion that multiple solutions is a means to assess mathematical creativity.

**Defining Beliefs**

Researchers have documented the role of belief in mathematics education (Cooney, 1985; Hart, 2002; Philipp, 2007; Schommer-Aikins, Duell, & Hunter, 2005; Seaman, 2005; Thompson, 1992). Although, much has been disputed about the matter, as Mewborn and Cross (2007) have pointed out, beliefs can be seen as attitudes and knowledge. Tann (1993) offers “personal theories” as ways of thinking about—a person’s set of beliefs, values, understanding, [and] assumptions” (p. 5). Hart (2002) describes beliefs as being “formed through experience over time” (p. 4). Howse (2006) defines beliefs as “one’s personal notions, perceptions, and conceptions of truth and reality” (p. 26). Functionally, for the purpose of this study, belief should be understood in these terms.

**Mathematical Beliefs**

In understanding how some view mathematics Mewborn and Cross (2007) contrast two differing beliefs. One way to dichotomize these beliefs is to see mathematics as fixed and the other view is to see mathematics as fluid. The fixed view would include the belief that:

- Mathematics is computation.
- Mathematics problems should be solved in less than five minutes, or else there is something wrong with either the problem or the student.
- The goal of doing a mathematics problem is to obtain the correct answer.
- In the teaching-learning process, the student is passive and the teacher is active.

Van De Walle (2007) describes this fixed belief of mathematics as the traditional view. Furthermore, it is believed from this perspective that mathematics is a collection
of rules to be mastered, arithmetic computations, mysterious algebraic equations and geometric proofs” (p. 12). That is to say, mathematics is dead to be examined like a corpse in an autopsy. He continues that the fixed position of mathematics believes that: mathematics is a series of arbitrary rules, handed down by the teacher, who in turn got them from some very smart source” (p. 12). When mathematics is believed to be fixed, doing mathematics is following some rule, knowing mathematics is applying the correct rule, and determining the correct answer is held by the expert, that is some teacher or book.

In contrast to the fixed belief of mathematics, according to Mewborn and Cross (2007), the fluid view (which corresponds to the National Council of Teachers of Mathematics position) would include the belief that:

- Mathematics is problem solving.
- Mathematics problems come in different types. Some can be solved quickly by recall. Other mathematics problems require a significant amount of time to understand the task, experience with possible solution methods, reach an answer, and check to see that the answer makes sense.
- The goal of doing a mathematics problem is to make sense of the problem, the solution process, and the answer.
- In the teaching-learning process, the student and teacher are both active in making sense of the mathematics and of students’ reasoning.

This fluid belief sees mathematics as a science of pattern and order” (Mathematics Sciences Education Board, 1989, p. 31). Essentially, mathematics is ever living and expanding. It can be discovered and explored where predictions and conjectures are made. Mathematics is a noun, but it is more than that. It includes the verbs (conjecture, discover, explore, investigate, predict, etc.) that conceive mathematics as well.

In short, the nature of mathematics can be seen as two poles. One belief is that mathematics is static. That is to say, mathematics is an arbitrary set of rules that are unchanging and uncompromising. And the other belief, which believes that mathematics is dynamic, sees mathematics as an ever growing, ever changing body of flux. Imitation or regurgitation is how mathematics can be seen in the static camp.
However, for those that see mathematics through the dynamic lens, it undertakes an active dimension of assimilation and creation.

**Defining Anxiety**

Anxiety has long been a research agenda for the social sciences (Trujillo & Hadfield, 1999). As a construct it is broadly defined, but generally speaking has the characteristics of fear and dread (Hembree, 1990). In practical terms it has been described as panic, tension, helplessness, fear, distress, shame, inability to cope, sweaty palms, nervous stomach, difficulty breathing, and loss of ability to concentrate (Trujillo & Hadfield, 1999). In more specific terms, May (1977) states, as cited in Hembree (1990), that anxiety is "the feelings of uncertainty and helplessness in the face of danger" (p. 33).

Like most construct, anxiety has been defined in different ways. Therefore, the researcher’s perspective determines how anxiety is defined. Anxiety, as a construct, has been viewed from either an affective or cognitive domain. That is to say, if anxiety is viewed from an affective position, then it refers to the emotional component of anxiety, feeling of nervousness, tension, dread, fear, and unpleasant physiological reactions to testing situations" (Ho, Senturk, Lam, Zimmer, Hong, Okamoto, Chiu, Nakazawa, & Wang, 2000, p. 363). However, on the other hand, if viewed from a cognitive platform, then it refers to the worry component of anxiety, which is often displayed through negative expectations, preoccupation with and self-deprecatory thoughts about an anxiety-causing situation" (p. 363).

Many researchers, in educational terms, are interested in test anxiety. Test anxiety may include the mental disorganization, disturbance, discomfort, or interference to perform on assessment or exams (Ma, 1999; Swars, Daane, & Giesien, 2006; Trujillo & Hadfield, 1999). Ho and colleagues describe test anxiety as reactions to testing situations with emotions and feelings of anxiety or nervousness which may even include tension, dread or fear (Ho et al, 2000).

**Mathematical Anxiety**

For several decades, anxiety has been studied within the content domain of mathematics. According to Hembree (1990) mathematics anxiety is reported to be, —no
more than subject-specific test anxiety,” while others have stated that it is basically, —a general dread of mathematics, and of tests in particular” (p. 34). Ma (1999) simply defines it as dislikes, worries, and fears towards mathematics. However, some researchers have acknowledged the complex nature of describing mathematics anxiety, because as a construct it possesses both affective and cognitive aspects (Sloan, Daane, & Giesien, 2003).

Mathematics anxiety has also been defined contextually. For instance Hopko (2003) illustrates it as, —aprehension and arousal concerning the manipulation of numbers in academic, private, and social environments” (p. 336). Other researchers have stressed that mathematics anxiety produces avoidance behaviors to mathematics as a stimulus (Ashcraft, 2002; Hopko, Hahadevan, Bare, & Hunt, 2003) In short, mathematics anxiety is an intricately complex and multidimensional construct (Sloan et al, 2003; Ma, 1999) which possesses a —state of discomfort that occurs when an individual is required to perform mathematically, or the feeling of tension, helplessness, or mental disorganization an individual has when required to manipulate numbers and shapes” (Swars, Daane, & Giesien, 2006).

An intriguing question remains who or what is responsible for producing mathematics anxiety? According to Trujillo and Hadfield (1999) causes of mathematics anxiety are environmental, intellectual (or cognitive), and personality factors. Similarly, Cemen (1987), as cited by Ma (1999), gives three reactions to produce anxious reactions. First, there are environmental precursors which are negative experiences at home or in the classroom with mathematics. Next, there are dispositional precursors, this may entail negative attitudes towards mathematics or a lack of confidence in it. Finally, there are situational precursors which are factors or formats of the classroom or its instruction. It is suggested by some (Sloan, Daane, & Giesien, 2003) that teachers who have high mathematics anxiety are likely to convey mathematics anxiety to their students. Trujillo and Hadfield (1999) have proposed a theoretical model for the causes of mathematics anxiety for pre-service teachers. They suggest that, —negative classroom experiences in mathematics and lack of support at home combined with an anxiety toward telling will yield a mathematically anxious individual” (Trujillo & Hadfield
1999, p. 227). Although teachers may not be the only catalyst for mathematics anxiety, they are an important factor.
CHAPTER 3
METHODOLOGY

Introduction

In this chapter the research design, sample population, treatment, and instruments will be discussed. The overarching goal of this study was to understand the relationships among several variables: mathematical creativity, mathematical beliefs, and mathematical anxiety. That is, how does mathematical creativity and mathematical beliefs interact, and how does mathematical creativity and mathematical anxiety relate to one another? This chapter will present a methodological approach to understanding these variables in this study. In short, this study employed a quasi counterbalance experimental design. Participants were pre- and post-tested to obtain scores for mathematical anxiety, mathematical beliefs, and mathematical creativity.

The following questions guided this study:

1. What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity?
2. What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs?
3. What relationship exists between elementary pre-service teacher’s mathematical creativity and their mathematical beliefs?
4. What relationship exists between elementary pre-service teacher’s mathematical creativity and mathematical anxiety?

The null hypothesis was the following: there is no correlation between mathematical creativity and one's epistemological belief about the nature of mathematics for elementary pre-service teachers at a research institution in the southeast region of the United States. It was predicted that individuals with higher creativity scores would possess an epistemological belief that the nature of
mathematics is more fluid than fixed. The null hypothesis was that there is no effect on one’s mathematical creativity when exposed to mathematical creativity for elementary pre-service teachers at a southeast regional institution. Individuals, who were exposed to mathematical creativity, it was expected, would have higher mathematical creativity scores. The null hypothesis was that there is no correlation between a punctuated, intentional experience to mathematical creativity and epistemological belief about the nature of mathematics for elementary pre-service teachers at a southeast regional institution. Furthermore, it was predicted that a punctuated, intentional experience to mathematical creativity would influence elementary pre-service teacher’s epistemological beliefs of the nature of mathematics. The null hypothesis was the following: there is no correlation between mathematical creativity and mathematical anxiety for elementary pre-service teachers at a research institution in the southeast region of the United States. Finally, it was predicted that mathematical creativity and mathematics anxiety would correlate.

Research Design

Pre-service teachers, during the fall semester of 2009, were pre- and post-tested with four different instruments. To assess their epistemological beliefs about the nature of mathematics, the Mathematics Belief Questionnaire (MBQ) was given (Collier, 1972). The Abbreviated Math Anxiety Scale (AMAS) instrument was used to measure the pre-service teachers’ mathematical anxiety (Hopko, Mahadevan, Bare, & Hunt, 2003). Using the following two different instruments, mathematical creativity was measured: Balka’s (1974a) Creativity Ability in Mathematics (CAMT) and Silver and Cai’s (2005) General Assessment Criteria (GAC).
Table 1
Counterbalance Design

<table>
<thead>
<tr>
<th>Pre Assessment</th>
<th>Post Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Group A</td>
<td>MBQ Treatment</td>
</tr>
<tr>
<td>randomize</td>
<td>AMAS</td>
</tr>
<tr>
<td></td>
<td>GAC</td>
</tr>
<tr>
<td>Group B</td>
<td>MBQ No treatment</td>
</tr>
<tr>
<td>randomize</td>
<td>AMAS</td>
</tr>
<tr>
<td></td>
<td>GAC</td>
</tr>
</tbody>
</table>

Next, two different groups were formed by randomly assigning the pre-service teachers. Using a random number table, pre-service teachers were either assigned to group A or B. Group A first received the treatment (a punctuated, intentional experience to problem posing, divergent thought and invented strategies) and group B did not receive the treatment. Both groups were assessed using the GAC. Then the treatment was switched. That is, group A did not receive the treatment, but group B did receive the treatment. Following the counterbalance, the two groups were assessed with the GAC. As stated earlier, to establish a baseline measurement at the beginning of the semester the pre-service teachers were assessed with GAC and then at the end of the semester the pre-service teachers were assessed with GAC again. (See Table 1 for the counterbalance design with the pre- and post-test.)

Setting and Sample

Procedures

For this study potential participants were recruited from an elementary mathematics methods course in the College of Education. Potential participants were given a general overview of the study and a third party person distributed and collected the adult consent forms from those who wished to participate. The consent forms were stored in a locked filing cabinet in a locked office.
Participants completed the two surveys for mathematical beliefs and mathematical anxiety; they were collected via internet. The information was stored in a remote database – Blackboard® – which was and is password protected. The other data on mathematical creativity were collected in a paper-and-pencil format. This data, like the consent forms, were and are stored in a locked office in a locked cabinet. The data will be destroyed after five years from the end of the study.

**Participants**

This study employed a convenient sampling method. The participants that were selected as the sample for this study were juniors entering into the elementary education program at a research institution in the southeast region of the United States. These pre-service teachers meet the university’s course requirements for mathematics (a minimum of six hours to includes college algebra or higher) and the College of Education’s prerequisites as well (which comprises passing the General Knowledge test and a minimum 2.5 grade point average). This sample of 32 students were studied in their first mathematics methods course MAE 4326, *How Children Learn Mathematics*. This sample was rather homogeneous with few minority students and overwhelmingly female. Specifically, the sample was comprised of one African American female and thirty-one white pre-service teachers (of which one was a white male).

Although the sample was not randomly selected, the participants were randomly assigned to one of two groups. Using a random number table beginning with the fifth row and the seventh column, the first sixteen appropriate two digit numbers in the table bifurcated the participants into these two groups.

**Treatment**

During the treatment, pre-service teachers participated in a ninety minute class session. In a fifteen week semester, the treatment was administered at the end of the second and the beginning of the third week of the semester to group A and B respectively. The following protocol was used to ensure the treatment was the same for both randomly assigned groups. The session cycled through a four-phase progression, which is the punctuated, intentional experience to mathematical creativity. The
progression of the four-phases was as follows: 1) exposure to multiple perspectives, 2) pose an open-ended problem, 3) examine sample solutions, and 4) pose alternative problems.

First, the pre-service teachers were exposed to multiple perspectives given four numbers, shapes, or objects. For an example see figure 4. They were asked questions like: Which one does not belong? Which one is different? What do they have in common? Which ones are the same? What is the pattern? Each question was pursued by the question Why? to justify the response. Not only were the pre-service teachers asked to formulate one rule and reason, but they were then asked to find at least two rules for each scenario. After the pre-service teacher responded to the prompt individually then they were given the opportunity to share their responses in groups and finally with the whole class (replicating a Think-Pair-Share model).

Figure 4. Example of an Multiple Perspective Task. Directions: Which One Does Not Belong? Why?

In the second phase, an open-ended problem was posed. The pre-service teachers were then given the opportunity to approach each problem from several vantage points. Time was given for them to explore the problem and to work it using several different methods or finding numerous solutions. For instance, the task, "Given
nine-dot unit-square grid draw as many shapes as possible with an area of 2 units²,” was adapted from Haylock (1997, p. 72).

Then sample solutions, in the third phase, were shown to the pre-service teachers of the open-ended problem. Time was given to discuss and understand the variegated solutions. The solutions exposed the pre-service teachers to creative thought. For the previous task, Haylock provided the following samples, from highly common constructions to highly creative creations. As seen in figure 5.

![Figure 5. Sample Works by Haylock, D. (1997). Recognizing mathematical creativity in school children. *International Reviews on Mathematical Education*, 29(3), 68-74.](image)

Finally, in the last phase, the pre-service teachers problem posed. Given the previous open-ended problems, they were asked to pose alternative problems that stem from the original problem and/or its solution. Changing the parameters or conditions of the problem were suggested.

**Instrumentation**

The epistemological beliefs about the nature of mathematics of pre-service teachers were measured (at the beginning and at the end of the semester) to determine their beliefs toward four specific domains: —mathematics is a collection of rules, formulas, and procedures; mathematics is a creative endeavor; mathematical problem-solving allows for multiple approaches; and mathematics is best taught by direct instruction” (Seaman, 2005, p. 6). This belief construct was measured with the *Mathematics Belief Questionnaire* developed by Collier (1972) and used by Seaman.
(2005) using a six point Likert scale (See Appendix A and Table 2). Each item had a scale response that ranges from strongly disagree to strongly agree. The MBQ’s reported reliability ranges from .80 and .83 using “the proportion of total variance that is not due to error in measurement” (Collier, 1972, p. 157). This survey was comprised of forty questions. Half of the items were posed in a positive direction and the other half in the negative direction. The positively stated items aligned with a fluid view of the nature, teaching, and learning of mathematics. Conversely, the fixed view of the nature, teaching, and learning of mathematics were identified with the negatively stated items.

These items can be understood using Seaman’s (2005) categorization into four general themes or domains: mathematics is a collection of rules, formulas, and procedures (items 1, 3, 9, 14, and 28); mathematics is a creative endeavor (items 2, 5, 6, 12, 18, 20, 23, 24, 33, 35, 37, and 39); mathematical problem solving allows for multiple approaches (items 4, 7, 8, 10, 13, 15, 19, 25, and 30); mathematics is best taught by direct instruction (items 21, 22, 26, 29, 31, 32, 34, 36, 38, 40).
### Mathematical Belief Questionnaire Organize by Seaman’s Four Categories

**Mathematics is a collection of rules, formulas, and procedures.**

1. Solving a mathematics problem usually involves finding a rule or formula that applies.
2. The main benefit from studying mathematics is developing the ability to follow directions.
3. In mathematics, perhaps more than in other fields, one can find set routines and procedures.
4. Math is an organized body of knowledge which stresses the use of formulas to solve problems.
5. Most exercises assigned to students should be applications of a particular rule or formula.

**Mathematics is a creative endeavor.**

2. The field of math contains many of the finest and most elegant creations of the human mind.
3. Studying mathematics helps to develop the ability to think more creatively.
4. The basic ingredient for success in mathematics is an inquiring nature.
5. In mathematics, perhaps more than in other areas, one can display originality and ingenuity.
6. Mathematics requires very much independent and original thinking.
7. The language of math is so exact that there is no room for variety of expression.
8. Children should be encouraged to invent their own mathematical symbolism.
9. Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error.
10. Math has so many applications because its models can be interpreted in so many ways.

**Mathematical problem solving allows for multiple approaches.**

4. The laws and rules of mathematics severely limit the manner in which problems can be solved.
7. There are several different but appropriate ways to organize the basic ideas in mathematics.
8. In mathematics there is usually just one proper way to do something.
9. There are several different but logically acceptable ways to define most terms in math.
10. Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.
11. There are often many different ways to solve a mathematics problem.
12. Each student should feel free to use any method for solving a problem that suits him or her best.
13. Teachers should frequently insist that pupils find individual methods for solving problems.
Table 2
Mathematical Belief Questionnaire Organize by Seaman’s Four Categories (Continued)

<table>
<thead>
<tr>
<th>Mathematics is best taught by direct instruction.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21. The teacher should always work sample problems for students before making an assignment.</td>
</tr>
<tr>
<td>22. Teachers should make assignments on just that which has been thoroughly discussed in class.</td>
</tr>
<tr>
<td>26. Teachers should provide class time for students to experiment with their own mathematical ideas.</td>
</tr>
<tr>
<td>29. Teachers should spend most of each class period explaining how to work specific problems.</td>
</tr>
<tr>
<td>31. Discovery methods of teaching have limited value because students often get answers without knowing where they came from.</td>
</tr>
<tr>
<td>32. The teacher should provide models for problem solving and expect students to imitate them.</td>
</tr>
<tr>
<td>34. The teacher should consistently give assignments which require research and original thinking.</td>
</tr>
<tr>
<td>36. Teachers must frequently give students assignments which require creative or investigative work.</td>
</tr>
<tr>
<td>38. Discovery-type lessons have very limited value when you consider the time they take up.</td>
</tr>
<tr>
<td>40. Students of all abilities should learn better when taught by guided discovery methods.</td>
</tr>
</tbody>
</table>

To measure mathematical anxiety, the pre-service teachers were pre- and post-tested using the *Abbreviated Math Anxiety Scale* (AMAS) at the beginning and the end of the semester. The survey had nine items. Each item was on a 5-point Likert scale, ranging from 1 (*low anxiety*) to 5 (*high anxiety*). The instrument’s internal consistency was reported to possess Cronbach’s alpha of .90 with a mean and standard deviation of 21.1 and 7.0 respectively (Hopko et al, 2003).

Mathematical creativity was measured using two different instruments. At the beginning and at the end of the semester they were tested using Balka’s (1974a) *Creative Ability in Mathematics Test* (CAMT) (See Appendix B). Balka (1974a) reported a Cronbach’s alpha of .72 and a standard error of measurement of 7.24 for CAMT reliability. The problems solved in class were assessed using Silver and Cai’s (2005) *General Assessment Criteria* (GAC) (See Appendix C). At four punctuated times, this scoring rubric was used to assess the pre-service teachers’ creativity.
Data Collection and Analysis

Various methods of data collecting were employed in this study of mathematical anxiety, mathematical beliefs, and mathematical creativity. To collect data on mathematical anxiety and mathematical beliefs, two different surveys were distributed electronically. As for mathematical creativity, that data were collected via the traditional means – paper-and-pencil.

The participants were asked to complete the Mathematics Belief Questionnaire and the Abbreviated Math Anxiety Scale on the internet. During the first week of class, participants were asked to log-on to Blackboard® and complete the forty item questionnaire for beliefs and the nine item survey for anxiety. The internet survey permitted the participants extra time to complete the surveys and the convenience of when and where to complete them as well.

Data on mathematical creativity were collected in class using the Creative Ability in Mathematics Test (CAMT) and the General Assessment Criteria (GAC). The participant were allowed the whole class period to respond to the CAMT at the beginning and the end of the semester. At punctuated times during the semester, participant were given four different items scored by the GAC. Each time fifteen minutes was given to the participant to complete the item.

To answer the first research question (What relationship exists between elementary pre-service teacher’s mathematical creativity and their mathematical beliefs?), the data (of epistemological beliefs of the nature of mathematics) from the MBQ were analyzed using a paired-samples t test. The null hypothesis was the following: there is no correlation between mathematical creativity and one’s epistemological beliefs about the nature of mathematics for elementary pre-service teachers at a research institution in the southeast region of the United States. Those factor scores of beliefs were then correlated with the mathematical creativity score from the CAMT. Mathematical creativity was a scale score derived from the CAMT scoring protocol. Regarding the answers to the first question (What relationship exists between mathematical creativity and elementary pre-service teacher’s epistemological beliefs of the nature of mathematics?), it was the author’s hypothesis that a positive correlation exist between fluid beliefs about the nature of mathematics and mathematical creativity.
These two scores were correlated twice, once at the pre-test and then again at the post-test.

Using a paired-samples \(t\) test, these two were compared to answer the third research question: What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs? The null hypothesis was that there is no correlation between a punctuated, intentional experience to mathematical creativity and epistemological beliefs about the nature of mathematics for elementary pre-service teachers at a southeast regional institution. However, it was the researcher’s hypothesis to find that a punctuated, intentional experience to mathematical creativity would affect pre-service teachers in viewing mathematics as more fluid than fixed.

To answer the second research question – What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity? – the data collected from the counterbalance design were analyzed using ANOVA. The null hypothesis was that there is no effect on one’s mathematical creativity when exposed to mathematical creativity for elementary pre-service teachers at a southeast regional institution. It was expected that a punctuated, intentional experience to mathematical creativity would foster mathematical creativity.

For the fourth research question (What relationship exists between elementary pre-service teacher’s mathematical creativity and mathematical anxiety?), scores were collected at the beginning and at the end of the semester, then a paired-samples \(t\) test was used to analyze and answer this question. The null hypothesis was the following: there is no correlation between mathematical creativity and mathematical anxiety for elementary pre-service teachers at a research institution in the southeast region of the United States. It was the researcher’s hypothesis that mathematical creativity scores would relate to mathematical anxiety scores.
CHAPTER 4
RESULTS

Introduction

This dissertation sought to understand mathematical creativity, mathematical beliefs, and mathematical anxiety. The present chapter will present the statistical data collected from the participants. To study mathematical creativity, mathematical beliefs, and mathematical anxiety, a quasi counterbalance experimental design was employed. Participants were pre- and post-tested to obtain scores for mathematical anxiety, mathematical beliefs, and mathematical creativity. The treatment was administered with the counterbalance experiment design.

Research Questions

The following research questions guided the inquiry:

1. What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity?
2. What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs?
3. What relationship exists between elementary pre-service teacher’s mathematical creativity and their mathematical beliefs?
4. What relationship exists between elementary pre-service teacher’s mathematical creativity and mathematical anxiety?

Research Findings

What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity?

A two-way within-subjects analysis of variance was conducted to evaluate the effect of a punctuated, intentional experience to mathematical creativity on elementary pre-service teacher’s mathematical creativity. The dependent variable was
mathematical creativity. The within-subjects factors were treatment groups. The mathematical creativity main effect and mathematical creativity x treatment groups effect were tested using the multivariate criterion of Wilk’s lambda ($\Lambda$). The mathematical creativity main effect and treatment group interaction effect, was significant, $\Lambda=.21, F (2, 6) = 51.09, p < .01$, as well as the mathematical creativity x treatment groups interaction effect, $\Lambda=.60, F (2, 6) = 9.06, p < .01$. One again, the results support the conclusion that a punctuated, intentional experience to mathematical creativity develops elementary pre-service teacher’s mathematical creativity as seen in table 3 and in figure 6.

Figure 6. Mathematical Creativity Scores During the Intervention
Table 3
Means (Standard Deviation) for General Assessment Criteria

<table>
<thead>
<tr>
<th></th>
<th>Mathematical Creativity Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time 0</td>
</tr>
<tr>
<td>Group A</td>
<td>12.57 (2.62)</td>
</tr>
<tr>
<td>Group B</td>
<td>14.56 (3.79)</td>
</tr>
</tbody>
</table>

What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity?

A paired-samples t test was conducted to evaluate mathematical creativity as to whether the means of the pretest was significantly different from the posttest. The results indicated that the pretest sample mean for mathematical creativity ($M = 35.13$, $SD = 10.56$) was significantly different from the posttest sample ($M = 40.24$, $SD = 11.42$), $t(31) = 19.99, p < .01$. The effect size of $d$ was 3.53. The 99% confidence interval for mathematical creativity mean ranged from 30.01 to 40.24 on the pretest and 34.81 to 45.88 on the posttest. Figure 7 shows the distribution of the mathematical creativity scores. The results support the conclusion that a punctuated, intentional experience to mathematical creativity increases or fosters elementary pre-service teacher’s mathematical creativity.
What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs?

A paired-sampled t test was conducted on the mathematical beliefs’ scores to evaluate whether the means of the pretest was significantly different from the posttest. The pretest sample mean 150.19 (SD = 15.77) was significantly different from the posttest sample mean 185.63 (SD = 21.14), \( t(31) = 49.67, p = .01 \). The 99% confidence interval for mathematical creativity mean ranged from 142.54 to 157.84 on the pretest and 175.37 to 195.88 on the posttest. The effect size of \( d \) was 8.78. Figure 8 shows the distribution of the mathematical beliefs scores. The results support the conclusion that a punctuated, intentional experience to mathematical creativity increases elementary pre-service teacher’s beliefs that mathematical is fluid.
Figure 8. Boxplots of Pre- and Post-Mathematical Beliefs Scores

*What relationship exists between elementary pre-service teacher’s mathematical creativity and their mathematical beliefs?*

Correlation coefficients were computed among the following two variables: mathematical creativity and mathematical beliefs. Although, a medium correlation coefficient appeared in the pretest correlation between mathematical creativity and mathematical beliefs scales, it was not significant $r = .256, p = .078$. At the same time, the correlation between mathematical creativity and mathematical beliefs scales for the posttest was not significant either $r = -.084, p = .324$. In general, the results suggest that no relationship exist between elementary pre-service teacher’s mathematical
creativity and their mathematical beliefs. In other words, mathematical beliefs are not a predictor of elementary pre-service teacher's mathematical creativity.

Table 4
Correlations Among the Pre- and Post- Scores for Mathematical Beliefs and Creativity

<table>
<thead>
<tr>
<th></th>
<th>Pre Fluid Mathematical Beliefs</th>
<th>Pre Mathematical Creativity</th>
<th>Post Fluid Mathematical Beliefs</th>
<th>Post Mathematical Creativity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre Fluid Mathematical Beliefs</td>
<td>1</td>
<td>.256</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Pre Mathematical Creativity</td>
<td>.256</td>
<td>1</td>
<td>.319*</td>
<td>1</td>
</tr>
<tr>
<td>Post Fluid Mathematical Beliefs</td>
<td>.319*</td>
<td>-.025</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Post Mathematical Creativity</td>
<td>.071</td>
<td>.281</td>
<td>-.084</td>
<td>1</td>
</tr>
</tbody>
</table>

*Note: N = 32.
*p < .005

**What relationship exists between elementary pre-service teacher's mathematical creativity and mathematical anxiety?**

Correlation coefficients were computed between mathematical creativity and mathematics anxiety scales. The results of the correlational analyses presented in Table 5 show that 1 out of the 6 correlations were statistically significant and were greater than or equal to .350. For the pretest, the correlation between mathematical creativity and mathematical anxiety scales was not significant \( r = .022, p = .453 \). The correlations of mathematical creativity with mathematics anxiety measures on the posttest tended to be lower and not significant. That is, the correlation between mathematical creativity and mathematical anxiety scales for the posttest was not significant either \( r = -.292, p = .052 \). In general, the results suggest that if mathematical creativity is higher then mathematics anxiety is lower (or vice versa).
Table 5
Correlations Among the Pre- and Post- Scores for Mathematical Anxiety and Creativity

<table>
<thead>
<tr>
<th></th>
<th>Pre Mathematical Anxiety</th>
<th>Pre Mathematical Creativity</th>
<th>Post Mathematical Anxiety</th>
<th>Post Mathematical Creativity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre Mathematical Anxiety</td>
<td>1</td>
<td>.022</td>
<td>1</td>
<td>-.223</td>
</tr>
<tr>
<td>Pre Mathematical Creativity</td>
<td>.022</td>
<td>1</td>
<td>-.223</td>
<td>1</td>
</tr>
<tr>
<td>Post Mathematical Anxiety</td>
<td>.587**</td>
<td>-.223</td>
<td>1</td>
<td>-.292</td>
</tr>
<tr>
<td>Post Mathematical Creativity</td>
<td>-.179</td>
<td>.281</td>
<td>-.292</td>
<td>1</td>
</tr>
</tbody>
</table>

*Note: N = 32.*  
**p < .001

However, a paired-sampled t test was conducted on the mathematical anxiety’ scores to evaluate whether the means of the pretest was significantly different from the posttest. The pretest sample mean 26.56 (SD = 4.85) was significantly different from the posttest sample mean 25.66 (SD = 5.78), t(31) = 25.12, p = .01. The 99% confidence interval for mathematical creativity mean ranged from 24.21 to 28.91 on the pretest and 22.85 to 28.46 on the posttest. The effect size of d was 4.44. Figure 9 shows the distribution of the mathematical anxiety scores. The results suggest that a punctuated, intentional experience to mathematical creativity decreases elementary pre-service teacher’s mathematical anxiety.
Figure 9. Boxplots of Pre- and Post-Mathematical Anxiety Scores
CHAPTER 5

DISCUSSION

Introduction

The purpose of this exploratory study was to examine how three particular variables (mathematical creativity, beliefs, and anxiety) relate to one another in pre-service teachers. Outlined in this chapter is a discussion of the results along with their implications. In addition, the limitations of this study will be discussed and suggestions for future research will be noted.

In studying mathematical creativity, beliefs, and anxiety, four questions directed and guided this research:

1. What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity?
2. What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs?
3. What relationship exists between elementary pre-service teacher’s mathematical creativity and their mathematical beliefs?
4. What relationship exists between elementary pre-service teacher’s mathematical creativity and mathematics anxiety?

The research literature strongly suggests that pre-service teachers, especially in elementary education, are likely to possess mathematical anxiety (Battista, 1986; Bursal & Paznokas, 2006; Gresham, 2007; Harper & Daane, 1998; Hembree, 1990; Vinson, 2001). Furthermore, mathematical beliefs have been associated with how one conceives the role of the teacher and student, how mathematics is taught and learned, and what instructional activities or problems should be used (Mewborn & Cross, 2007). As for mathematical creativity, it is a sparsely explored arena in the research fields of education.
This study employed a counterbalance design, randomizing the class into two groups and giving a pre- and post-test to determine if significant differences existed in the participants who were treated to a four-phase progression that was a punctuated, intentional experience of mathematical creativity in the form of problem posing, divergent thought, and invented strategies. This difference was also gauged using repeated measures during the experiment. Furthermore, beliefs and anxiety were correlated with mathematical creativity employing pre- and post-test measures.

Discussion

Mathematical creativity, although it has no universally agreed upon definition, is often measured or spoken of with three common elements. These elements are fluency, flexibility, and originality. Fluency is the quantity of acceptable responses to a given prompt. The number of different categories in the responses is flexibility. Although it is rather self-explanatory, originality refers to the novelty or the uniqueness of a response. For this study, mathematical beliefs have been defined by two bifurcated terms: fixed or fluid. The fixed belief of mathematics views the discipline as static and it is learned passively. On the other hand, the fluid belief of mathematics understands the body of knowledge to be dynamic (every growing and changing) and learning the discipline requires active participation. Angst towards mathematical situations or nervousness about scenarios pertaining to the context of school mathematics has been defined as mathematical anxiety.

Using a cognitive perspective to frame this study, *How People Learn: Bridging Research and Practice* (NCR, 1999), three major tenets were brought to the foreground. First, learners possess pre-existing knowledge. Second, the knowledge of the learner is multifaceted and contains a procedural, conceptual, and problem solving component. Third, metacognition (reflecting on conduct and content) plays a crucial role in the learner’s development.

Aspects of all three previous cognitive tenets were employed in the four-phase progression treatment – a punctuated, intentional experience with mathematical creativity. Drawing upon prior knowledge and experiences in the first phase (exposure to multiple perspectives) pre-service teachers related and reflected on new perspectives
and how they connected to former understandings. In the second phase (pose an open-ended problem) pre-service teachers solved mathematical tasks, which is problem solving with relational and procedural knowledge. Metacognition is required of pre-service teachers in the third phase (examine sample solutions) to reflectively understand how and why certain solution work for the given task. Finally, in the last phase (pose alternative problems) pre-service teachers were called to synthesize the previous three phases and extended mathematical tasks and scenarios in the activity of problem posing.

**What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity?**

To answer the question, **What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical creativity**, this study noted two interesting findings. First, using the counterbalance quasi-experimental design and an ANOVA data analysis, the results suggest that mathematical creativity can be developed or fostered. Second, after examining the CAMT pre- and post-test data, the mathematical creativity scores significantly increased, which further support this notion.

For the counterbalance quasi-experimental design, a two-way within-subjects analysis of variance was conducted to evaluate the effect of a punctuated, intentional experience to mathematical creativity on elementary pre-service teacher’s mathematical creativity. The dependent variable was mathematical creativity. The within-subjects factors were treatment groups. The mathematical creativity main effect and mathematical creativity x treatment groups effect were tested using the multivariate criterion of Wilk’s lambda ($\Lambda$). The mathematical creativity main effect and treatment group interaction effect, was significant, $\Lambda=.21, F (2, 6) = 51.09, p < .01$, as well as the mathematical creativity x treatment groups interaction effect, $\Lambda=.60, F (2, 6) = 9.06, p < .01$. 
The import of the following data must be stressed. First, note at time zero that
group A’s initial mean score was lower that group B’s mean score, as seen in figure 10.
Then group A received the treatment, while group B did not. Both were assessed at
time one. Significant gains were accrued to group A’s mean score by nearly double,
where as the mean score of group B made no significant change. Next group B
received the treatment and group A did not. At time two, the mean score for group B
increased significantly. It should also be noted at that time that group A’s mean score
slightly decreased. Nevertheless, the first treatment group started with a lower mean
score and finished with a larger mean score, even with a modest decline at the end of
time two.

This data suggest that a punctuated, intentional experience with mathematical
creativity increases elementary education pre-service teacher’s mathematical creativity.
With the tapering mean score of group A at time two, it may lead one to inquire if
prolonged, intentional experiences with mathematical creativity are required to maintain
these gains. To phrase it differently, does mathematical creativity atrophy and diminish
over time or in certain impoverished environments when it is not exercised?

Not to overstate what the data revealed, but the importance of this suggest that
pre-service teacher’s mathematical creativity can be significantly enhanced in a
relatively short period of time. If this is the case, how might this translate into school
settings in the mathematics classroom? Could it be that punctuated, intentional
experiences with mathematical creativity have the potential to redesign the landscape of the mathematics classroom?

Figure 10. Mathematical Creativity Scores During the Intervention

Furthermore, perhaps to extend the principle of this finding to school mathematics, could it not point to the need for continual, prolonged experiences with mathematical creativity? For instance, if a third grade student made gains in mathematical creativity, but the next year was not exposed to an environment that fostered mathematical creativity, would not the student’s mathematical creativity atrophy and diminish by the end of the fourth grade year? This hypothetical scenario solicits a solution which requires mathematics teachers at all levels or grades to afford students the opportunities to intentional experiences with mathematical creativity.
Then using the CAMT pre- and post-test data, a paired-samples t test was conducted to evaluate mathematical creativity as to whether the means of the pretest was significantly different from the posttest. The results indicated that the pretest sample mean for mathematical creativity ($M = 35.13$, $SD = 10.56$) was significantly different from the posttest sample ($M = 40.24$, $SD = 11.42$), $t(31) = 19.99$, $p < .01$. The effect size of $d$ was 3.53. The 99% confidence interval for mathematical creativity mean ranged from 30.01 to 40.24 on the pretest and 34.81 to 45.88 on the posttest.

Sriraman (2005) espouses that mathematical creativity and giftedness can be harmonized at the K-12 level using five principles: The Uncertainty Principle, The Scholarly Principle, The Free Market Principle, The Gestalt Principle, and The Aesthetic Principle. The current study at hand suggests that a punctuated, intentional experience to mathematical creativity (problem posing, divergent thinking, alternative algorithms, and invented strategies), corresponds to many of Sriraman’s five principles. By fostering an environment that tolerates ambiguity through open-ended or ill-posed problems the Uncertainty Principle is supported. The Scholarly Principle states that creativity challenges existing trains of thoughts and extends current knowledge. Creativity thrives where risk taking is encouraged and atypical or divergent thinking is promoted. The Gestalt Principle contends that with the freedom of time and movement creativity can flourish. This is the Free Market Principle. To behold solutions, methods, problems, or ways of thinking as objects of beauty is the Aesthetic Principle. In part, this study has maintained these five principles and the data suggest that mathematical creativity has been developed.

**What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs?**

A paired-sampled t test was conducted on the mathematical beliefs’ scores to evaluate whether the means of the pretest was significantly different from the posttest. The pretest sample mean 150.19 (SD = 15.77) was significantly different from the posttest sample mean 185.63 (SD = 21.14), $t(31) = 49.67$, $p = .01$. The 99% confidence interval for mathematical creativity mean ranged from 142.54 to 157.84 on the pretest and 175.37 to 195.88 on the posttest. The effect size of $d$ was 8.78.
Table 7
Summary Findings on Mathematical Beliefs When Exposed to Mathematical Creativity

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Instrument</th>
<th>Pre-Test Score</th>
<th>Post-Test Score</th>
<th>Results</th>
<th>Statistical Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>What effect does a punctuated, intentional experience to mathematical creativity have on elementary pre-service teacher’s mathematical beliefs?</td>
<td>Mathematics Belief Questionnaire</td>
<td>150.19 (15.78)</td>
<td>185.63 (21.14)</td>
<td>Significant</td>
<td>t test</td>
</tr>
</tbody>
</table>

In general, beliefs are hard to change (Murphy & Mason, 2004; Pajares, 1992; Thompson, 1992). Nevertheless, beliefs can be permeated under certain conditions. As Murphy and Mason state (2004), “although changes [in beliefs] can occur by chance, serendipity, without awareness, only high levels of cognitive, metacognitive, and motivational engagement lead to deeper and longer lasting change” (p. 311). In the present study, beliefs about mathematics did change. At the commencement of the study, the students mean average score was 150.19 on a scale ranging from 40 to 240. Consequentially, at the conclusion of the study, the mean average score of the students was 185.63. This results suggests that a punctuated, intentional experience to mathematical creativity effects elementary pre-service teacher’s mathematical beliefs. Analogous to this finding, Harts (2002) considered problem situation or dilemmas to change beliefs while examining alternative and invented algorithms. Silver (1994) argues that “one needs to understand the activities or practice of persons who are makers of mathematics” (p. 22). Could it be that understanding the cognitive process to differing methods to mathematical problems is perhaps a genesis or catalyst to changing the beliefs that mathematics is not fixed but fluid in nature?

**What relationship exists between elementary pre-service teacher’s mathematical creativity and their mathematical beliefs?**

Although, the results of Schommer-Aikins and colleagues (2005) suggest that beliefs factor into problem solving performance, in this current study, beliefs were not
directly linked to mathematical creativity, which is sometimes viewed as a subcategory of problem solving. In a qualitative study, Sriraman (2004) found that beliefs regarding the nature of mathematics played a role into how mathematical creativity was intricately involved. Even though the pre-service teachers in the present study increased their fluid or informal view of mathematics, it did not correlate with mathematical creativity. Note, the two variables that were measured correlated at the beginning and end of the study. Neither of the correlations were significant.

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Instrument</th>
<th>Pre-Test Score</th>
<th>Post-Test Score</th>
<th>Results</th>
<th>Statistical Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>What relationship exists between elementary pre-service teacher's mathematical creativity and their mathematical beliefs?</td>
<td>Creative Ability in Mathematics &amp; Mathematics Belief Questionnaire</td>
<td>.256 (.078)</td>
<td>-.084 (.324)</td>
<td>Not Significant</td>
<td>correlation</td>
</tr>
</tbody>
</table>

**What relationship exists between elementary pre-service teacher's mathematical creativity and mathematics anxiety?**

Haylock (1987b) confirmed his hypothesis that the highly mathematically creative students would have lower anxiety compared to their contemporaries. On the contrary, the present study could neither confirm nor deny such claims. The data did not prove to be significant. Differing instruments (for measuring mathematical creativity and anxiety were used) and sample populations, however, were considered in both studies. In spite of that, the current study suggested that a punctuated, intentional experience to mathematical creativity lowered mathematical anxiety. It should be noted that for the post-test, mathematical creativity and anxiety possessed a slight correlation ($r = -.292$ and $p = .052$). If this finding were significant, it would suggest that higher mathematical creativity scores would correlation with lower mathematical anxiety scores.
Table 9
Summary Findings on Mathematical Creativity and Mathematical Anxiety

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Instrument</th>
<th>Pre-Test Score</th>
<th>Post-Test Score</th>
<th>Results</th>
<th>Statistical Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>What relationship exists between elementary pre-service teacher’s mathematical creativity and mathematical anxiety?</td>
<td>Creative Ability in Mathematics &amp; Abbreviated Math Anxiety Scale</td>
<td>.022 (.453)</td>
<td>-.292 (.052)</td>
<td>Not Significant</td>
<td>correlation</td>
</tr>
</tbody>
</table>

**Implications**

The findings of this study point to three particular implications. First, if intentional experiences with mathematical creativity (such as problem posing, divergent thought, and invented strategies) change the view of elementary pre-service teacher’s beliefs about mathematics, which aligns more with NCTM’s (2000) vision, then perhaps teachers of mathematics, mathematics educators, and curriculum developers might consider integrating mathematically creative notions into their professional domains.

Second, if a punctuated, intentional experience to mathematical creativity fosters mathematical creativity, then conceivably this is a means to maintain and develop mathematical capital. Third, although, mathematical anxiety appears to be pandemic for nearly all levels of mathematics classes, intentional experiences with mathematical creativity may, in part, lowers mathematical anxiety for some at these levels.

Changing elementary education pre-service teacher’s mathematical beliefs and creativity is important. Conforming to the vision for school mathematics cast by NCTM (2000) under a problem solving motif, pre-service teachers, the nation’s future teachers, affords the next generation of students to experience the creative nature of mathematical problem solving. As state by NCTM (1980), "Problem solving, which is essentially a creative activity, cannot be built exclusively on routine, recipes, and formulas" (p. 4). These future teachers wield the potential power to positively influence students for years to come in the belief that mathematics is in fact a creative enterprise.
It has been indicated before that students at a young age need to be exposed to rich and robust situations that flow from real life problem solving situations to develop mathematical creativity (Sriraman, 2005). Admittedly, this goes without saying, for this to happen elementary mathematics school teachers must know of those situations and the deep mathematics behind those scenarios for the development of mathematical creativity. In like manner, mathematics educators must forge a foundation for the equipping of elementary teachers of mathematics to rise to this challenge.

By removing rigid fixations or mental blocks, it appears that creativity can be developed. As Sternberg and Lubart (1999) acknowledges, some people are mono-functional when it comes to certain tasks. That is, there is only one correct way to complete the task. However, if a task can be seen as multi-functional (or problems can be solved in a variety of ways), then perhaps such fixations and blockades can be bypassed to develop creativity. This line of thought hints at the vision and standards NCTM casts for classroom mathematics. For instance, an amalgamation of the process standards (e.g. problems solving, connections, and representation) might assist this notion of dismantling some fixations and facilitating or accessing new pathways.

In an effort to validate the four-stage Gestalt model (preparation, incubation, illumination, and verification) proposed by Hadamard (1945), Sriraman (2004) contends for its validity for current understanding of mathematical creativity. Furthermore, in the mathematics classroom, he argues for students’ engagement for prolonged periods of time with non-routine tasks (such as problem posing and open-ended or ill-posed problems) that require motivation, persistence, and most importantly reflection (Sriraman, 2004). In framing the question to mathematics teachers, how do we teach our students to —do mathematics? Parr (1974) contends that it is by using the four stages.

However, understanding the limitations and time constraints of school mathematics, she provided four strategies to support the four stages of creativity. First, the teacher should expect and require students to make conjectures about mathematical scenarios. This means, secondly, that the teacher and classroom should sustain a supportive classroom atmosphere. Third, when appropriate, the teacher should shorten the discovery process for natural limitations. Finally, because
mathematics is inherently beautiful, mathematics should be discussed and evaluated based on its aesthetics value. Is the proposed mathematics, in other words, a novel solution to a mathematical task, an alternative method to a mathematical dilemma, or a unique problem posed from a given mathematical situation? These questions and others should be integrated into the fabric of classroom discourse.

Discussing methods to foster creativity, Hashimoto (1997) suggests two broad methods. First, he calls for an open-ended approach. Within the open-ended approach, he catalogs three problem types: finding, classifying, and measuring. The finding problem type presents a given scenario and students are asked to find as many rules or relationships as possible. Given a collection of equations, figures, graphs, or items, student are asked to sort or classify them into different groups. This is an example of classifying problems types. Measuring problem types introduce a situation with the demands to measure a concept with ambiguous parameters. Second, Hashimoto (1997) calls for the method —From Problem to Problem” (p. 86). This second methods is analogous to problem posing and problem formulating. Once a problem is solved in one context, then the problem is extended in one way or another by changing the original problem’s conditions.

Practically speaking in the mathematics classroom, problem tasks and situations need to be presented that allow for ambiguity to foster mathematical creativity. One way this could be accomplished is by investigating growth patterns. Students could formulate conjectures for particular steps in the pattern, and then they could invent rules to predict, in general, any step in the pattern. However, the key is to examine the student’s perception of how the growth pattern changes and what remains the same. In many cases, because students’ perspectives are inherently different on the situation, the rule making activity often permits ambiguity and solicits opportunities for students to express their creative abilities in mathematics.

Corresponding to rule making activities, another ambiguous task for students is relationship finding. Given a collection of items, students are asked to sort the items based on some rule they devise. Once students have described their rule and the collection of items have been sorted or classified into two or more categories, ask them to resort the items again but based on a different relationship. Just to name a few
collections, this activity could use inanimate objects, geometric shapes, sets of numbers, and various functions with one or more representations (i.e. table, graph, equation, model, and verbal expression). By discussing how and why they sorted the related items affords the students an opportunity to express or experience creative thinking.

As previously mentioned, creativity is often stifled because of fixations. For instance, problem situations may be read or perceived in one narrow light, or the solution methods may be seen as a singular path. However, to erode these fixed views on mathematical problems, teachers ask students to work in groups and devise to two or more solution methods for the problem at hand. By forcing student to find multiple methods to mathematical situations, divergent thinking is being evoked. This may include requiring students to explore solutions using multiple representations. For example, if the students solved the problem situation using a table, ask them to find the solution again but this time with a graph, a model, or even an equation. Another way that divergent thought can overcome fixations is to ask student to analyze the work of others when an alternative algorithm or a non-routine method is employed to solve the problem. Understanding differing perspectives, along with using multiple methods and representations, are tangible means for teachers to increase students' fluid view of mathematics and decrease content and process fixations in mathematics.

**Limitations**

As for the ability to generalize the findings of this study, it would be careless to do so based upon the uniqueness of the sample of elementary education pre-service teachers. At any rate, the findings would seem likely to characterize other elementary education pre-service teachers with similar backgrounds and experiences. Nevertheless, in a broader sense this study with its findings contributes to the understanding of mathematical creativity, beliefs, and anxiety.

Several limitations exist for this present study, and many questions arise. One limitation might be that the study used a small sample size (N = 32). Additionally, as mentioned earlier, this sample was predominately white middle class females and exclusively elementary education pre-service teachers. Naturally one may inquire how
would a larger sample and how would middle or high school mathematics pre-service teacher compare to this sample. Furthermore, does gender matter and if so to what extent?

The researcher in this study was an active participant in the entire process who implemented the four-phase progression treatment to both groups. With particular beliefs about the nature of mathematics being dynamic and practical pedagogical ideologies about teaching and learning of mathematics that students must be actively engaged while constructing knowledge, the influence and effect of the participatory researcher as the teacher may account for some of the results in this study. Which is to say, the administering of a punctuated, intentional experience to mathematical creativity may not be replicable for other teachers with differing philosophical and practical beliefs. Modeling multiple methods and solutions, encouraging alternative algorithms, and discussing divergent thinking to mathematical tasks were innate to the teaching researcher of this study. In short, the mathematical environment, because of the personal factors of the researcher, may be an uncontrolled limitation to this study.

Following the same line of thought, the researching teacher has been described as enthusiastic and passionate about the teaching and learning of mathematics. Could it be that exposure to the teacher and not to the treatment may account for some of the results of this study? This is a real potential and alternative understanding of the current findings.

Another possible limitation could be that the design was not purely experimental with a control and treatment group. Rather, it was a counterbalance quasi-experimental design, with the strength to study an intact group, which controls for subjects characteristics and attitudes (Fraenkel & Wallen, 2009). In future experimental studies, a factorial design might be considered using differing levels of treatments and categories of participants.

Using the AMAS, data were collected on mathematical anxiety for the study. The AMAS is a survey with nine items using a Likert scale from one to five. The brevity of the instrument may have limited the variance in the score. Perhaps this accounts for the incongruity with Haylock’s (1985) findings that used the Mathematics Attitude Inventory by Wallach and Kogan (1965). This could imply that some instruments might
discern a relationship between the two variables of mathematical creativity and mathematical anxiety than other instruments.

Suggestions

If creativity (and more specifically mathematical creativity) can be developed or fostered, what activities aid in this endeavor? Conversely, what activities are counterproductive? In short, what pedagogical practices cultivate or encumber mathematical creativity? Romey (1970) contends that without creative teaching there will be no creative learning. Does this mean that creative learning is a by-product of creative teaching (and if so, then to what extent)? The import, from Romey's perspective, is that the entire enterprise of teaching must be creative: from curriculum mapping to problem posing, from lesson planning to lesson implementation, from mathematical activities to drill and practice, from classroom discourse to content evaluation (1970). Future studies, then, may consider what affect does creative teaching has on students' mathematical creativity.

In an earlier work, Banghart and Spraker (1962) reports that creativity in mathematics was not influenced by group activities. However, in Promoting Mathematical Creativity in the Classroom, Borenson (1981) describes a kind of communal creativity. In recent years, additionally, it has been suggested that group activities like brainstorming could enhance or generate creative responses (Nickerson, 1999). To frame the question, one could try to reconcile these two incongruities with this inquiry: what group practices stimulate curiosities that produce creative responses in mathematics?

To answer the question how can creativity be enhanced, one of Nickerson's (1999) suggestions is to stimulate curiosity. In the mathematics classroom, for one example, curiosity might be aroused by carefully observing some phenomenon that demonstrates some mathematical principle. Second of all, as Nickerson (1999) notes, —curiosity is contagious" (p. 410), which might mean that teachers possess an honest level of curiosity for mathematics, if this is to be effective. Finally, teachers of mathematics should invoke and foster wonder and awe to their students about the world in which they live, especially as to how mathematics models the universe as we know it.
At any rate, the impetus is to better understand the connection between mathematical creativity and curiosity, and to investigate if the two are content domain specific.

The conviction held by Hashimoto (1997) is that beliefs about mathematics held by teachers are connected to fostering students’ mathematical creativity. In future studies, this conjecture about the influence of teacher beliefs and its relationship to fostering mathematical creativity in students should be explored. Presumably, one could investigate the following question: Do particular epistemological views of mathematics and mathematically curious disposition held by the teacher influence students’ mathematical curiosity and creativity?

In this current study, to some extent, punctuated experiences with mathematical creativity were examined; it may behoove future studies to examine the effect of prolonged experiences with mathematical creativity. Furthermore, the treatment was intentional and explicit with the four-phase progression; would similar results be produced if the treatment was intentional but implicit using a different format that was deliberate but covert? Simultaneously examining and comparing elementary, middle, and secondary pre-service teachers while administering a punctuated, intentional experience with mathematical creativity would further our understanding on fostering mathematical creativity. Also, in more practical terms, do pre-service teachers with higher mathematical creativity integrate these notions into the planning of lessons? If so, how is manifested and accomplished?

**Conclusion**

The expectation was sustained by the data that a punctuated, intentional experience to mathematical creativity would foster mathematical creativity. It was predicted and supported by the study that a punctuated, intentional experience to mathematical creativity would affect pre-service teachers in viewing mathematics as more fluid than fixed. In addition, the data suggest that a punctuated, intentional experience to mathematical creativity lowers mathematical anxiety. However, it was not substantiated that either mathematical beliefs nor mathematical anxiety predicted mathematical creativity as conjectured by the researcher.
The findings of this study point to three particular implications. A punctuated, intentional experience to mathematical creativity, first, appears to change the perspective of pre-service teachers’ beliefs about mathematics, which aligns with NCTM’s (2000) vision for mathematics. Second, if a punctuated, intentional experience to mathematical creativity fosters mathematical creativity, then conceivably this is a means to maintain and develop mathematical capital and prowess. Third, although, mathematical anxiety appears to be pandemic for nearly all levels of mathematics classes, a punctuated, intentional experience to mathematical creativity may, in part, lowers mathematical anxiety for some at these levels.

In conclusion, intentional experiences with mathematical creativity provide hope. It potentiates change in the mind that mathematics is, in fact, a creative endeavor. Therefore, mathematics can and should be approached with alternative algorithms, differing representations, invented strategies, problem posing, and multiple methods based upon the learner’s prior knowledge and experiences. Transforming mathematical beliefs into a problem solving paradigm that coincides with NCTM’s vision may also provide an avenue to alleviate mathematical anxiety. Could it be that mathematical capital is harvested through these intentional experiences with mathematical creativity? How might the mathematic educators of today change the course of mathematical learning for the next generation? This study has suggested that there can be hope to foster mathematical creativity, beliefs can be bent toward NCTM’s vision for the mathematics classroom, and mathematic anxiety can be alleviated.
# MATHEMATICAL BELIEFS QUESTIONNAIREE

Please circle the number which best describes your agreement with each statement.


<table>
<thead>
<tr>
<th></th>
<th>Strongly disagree</th>
<th>Moderately disagree</th>
<th>Slightly disagree</th>
<th>Slightly agree</th>
<th>Moderately agree</th>
<th>Strongly agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Solving a mathematics problem usually involves finding a rule or formula that applies.</td>
<td>1</td>
<td>2</td>
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<td>5</td>
<td>6</td>
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<tr>
<td>2. The field of math contains many of the finest and most elegant creations of the human mind.</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<tr>
<td>3. The main benefit from studying mathematics is developing the ability to follow directions.</td>
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<tr>
<td>4. The laws and rules of mathematics severely limit the manner in which problems can be solved.</td>
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<tr>
<td>5. Studying mathematics helps to develop the ability to think more creatively.</td>
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<td>6</td>
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<tr>
<td>6. The basic ingredient for success in mathematics is an inquiring nature.</td>
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<td>6</td>
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<tr>
<td>7. There are several different but appropriate ways to organize the basic ideas in mathematics.</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<td>5</td>
<td>6</td>
</tr>
<tr>
<td>8. In mathematics there is usually just one proper way to do something.</td>
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<td>2</td>
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<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>9. In mathematics, perhaps more than in other fields, one can find set routines and procedures.</td>
<td>1</td>
<td>2</td>
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</tr>
<tr>
<td>10. Math has so many applications because its models can be interpreted in so many ways.</td>
<td>1</td>
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<tr>
<td>11. Mathematicians are hired mainly to make precise measurements and calculations for scientists.</td>
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<td>6</td>
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<tr>
<td>12. In mathematics, perhaps more than in other areas, one can display originality and ingenuity.</td>
<td>1</td>
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<tr>
<td>13. There are several different but logically acceptable ways to define most terms in math.</td>
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<tr>
<td>14. Math is an organized body of knowledge which stresses the use of formulas to solve problems.</td>
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<tr>
<td>15. Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.</td>
<td>1</td>
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<tr>
<td>16. Mathematics is a rigid discipline which functions strictly according to inescapable laws.</td>
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<tr>
<td>17. Many of the important functions of the mathematician are being taken over by the new computers.</td>
<td>1</td>
<td>2</td>
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<tr>
<td>18. Mathematics requires very much independent and original thinking.</td>
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<td>---</td>
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<td></td>
</tr>
<tr>
<td>19.</td>
<td>There are often many different ways to solve a mathematics problem.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>20.</td>
<td>The language of math is so exact that there is no room for variety of expression.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>21.</td>
<td>The teacher should always work sample problems for students before making an assignment.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>22.</td>
<td>Teachers should make assignments on just that which has been thoroughly discussed in class.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>23.</td>
<td>Children should be encouraged to invent their own mathematical symbolism.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>24.</td>
<td>Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>25.</td>
<td>Each student should feel free to use any method for solving a problem that suits him or her best.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>26.</td>
<td>Teachers should provide class time for students to experiment with their own mathematical ideas.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>27.</td>
<td>Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>28.</td>
<td>Most exercises assigned to students should be applications of a particular rule or formula.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>29.</td>
<td>Teachers should spend most of each class period explaining how to work specific problems.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>30.</td>
<td>Teachers should frequently insist that pupils find individual methods for solving problems.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>31.</td>
<td>Discovery methods of teaching have limited value because students often get answers without knowing where they came from.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>32.</td>
<td>The teacher should provide models for problem solving and expect students to imitate them.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>33.</td>
<td>The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>34.</td>
<td>The teacher should consistently give assignments which require research and original thinking.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>35.</td>
<td>Teachers must get students to wonder and explore even beyond usual patterns of operation in math.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>36.</td>
<td>Teachers must frequently give students assignments which require creative or investigative work.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>37.</td>
<td>Students should be expected to use only those methods that their text or teacher uses.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>38.</td>
<td>Discovery-type lessons have very limited value when you consider the time they take up.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>39.</td>
<td>All students should be required to memorize the procedures that the text uses to solve problems.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>40.</td>
<td>Students of all abilities should learn better when taught by guided discovery methods.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
*APPENDIX B*

**CREATIVE ABILITY IN MATHEMATICS TEST**

Name:________________________________________________________________________

Grade:__________ Age:__________ Boy or Girl? __________

**Directions**

The items in the booklet give you a chance to use your imagination to think up ideas and problems about mathematical situations. We want to find out how creative you are in mathematics. Try to think of unusual, interesting, and exciting ideas – things no one else in your class will think of. Let your mind go wild in thinking up ideas.

You will have the entire class time to complete this booklet. Make good use of your time and work as fast as you can without rushing. If you run out of ideas for a certain item go on to the next item. You may have difficulty with some of the items; however, do not worry. You will not be graded on the answers that you write. Do your best!

Do you have any questions?

**DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
**ITEM I**

**Directions**

Patterns, chains, or sequences of numbers appear frequently in mathematics. It is fun to find out how the numbers are related. For example look at the following chain:

2 5 8 11 ___ ___

The difference between each term is 3; therefore, the next two terms are 14 and 17. Now look at the chain shown below and supply the next three numbers.

1 1 2 3 5 8 13 21 ___ ___ ___

**ITEM II**

**Directions**

Below are figures of various polygons with all the possible diagonals drawn (dotted lines) from each vertex of the polygon. List as many things as you can of what happens when you increase the number of sides of the polygon. For example: The number of diagonals increases. The number of triangles formed by the number of diagonals increases.

1. ________________________________________________________________________
2. ________________________________________________________________________
3. ________________________________________________________________________
4. ________________________________________________________________________
5. ________________________________________________________________________
6. ________________________________________________________________________
7. ________________________________________________________________________
8. ________________________________________________________________________
9. ________________________________________________________________________
10. _______________________________________________________________________
11. _______________________________________________________________________
12. _______________________________________________________________________


ITEM III

Directions

Suppose the chalkboard in your classroom was broken and everyone’s paper was thrown away; consequently, you and your teacher could not draw any plane geometry figures such as lines, triangles, squares, polygons, or any others. The only object remaining in the room that you could draw on was a large ball or globe used for geography. List all the things which could happen as a result of doing your geometry on this ball. Let your mind go wild thinking up ideas.

For example: If we start drawing a straight line on the ball, we will eventually end up where we started. (Don’t worry about the maps of the countries on the globe.)

1. ________________________________________________________________________
2. ________________________________________________________________________
3. ________________________________________________________________________
4. ________________________________________________________________________
5. ________________________________________________________________________
6. ________________________________________________________________________
7. ________________________________________________________________________
8. ________________________________________________________________________
9. ________________________________________________________________________
10. ________________________________________________________________________
11. ________________________________________________________________________
12. ________________________________________________________________________
13. ________________________________________________________________________
14. ________________________________________________________________________
15. ________________________________________________________________________
16. ________________________________________________________________________
17. ________________________________________________________________________
18. ________________________________________________________________________
19. ________________________________________________________________________
20. ________________________________________________________________________
21. ________________________________________________________________________
22. ________________________________________________________________________
ITEM IV

Directions
Write down every step necessary to solve the following mathematical situation. Lines are provided for you to write on; however there may be more lines than you actually need.

Suppose you have a barrel of water, a seven cup can, and an eight cup can. The cans have no markings on them to indicate a smaller number of cups such as 3 cups. How can you measure nine cups of water using only the seven cup can and the eight cup can?

____________________________________________________________________________
____________________________________________________________________________
____________________________________________________________________________
____________________________________________________________________________
____________________________________________________________________________
____________________________________________________________________________

ITEM V

Directions
Suppose you were given the general problem of determining the names or identities of two hidden geometric figures, and you were told that the two figures were related in some manner. List as many other problems as you can which must be solved in order to determine the names of the figures. For example: Are they solid figures such as a ball, a box, or a pyramid? Are they plane figures such as a square, a triangle, or a parallelogram? If you need more space, write on the back of this page.

1._________________________________________________________________________
2._________________________________________________________________________
3._________________________________________________________________________
4._________________________________________________________________________
5._________________________________________________________________________
6._________________________________________________________________________
7._________________________________________________________________________
8._________________________________________________________________________
9._________________________________________________________________________
10._________________________________________________________________________
ITEM VI

Directions

The situation listed below contains much information involving numbers. Your task is to make up as many problems as you can concerning the mathematical situation. You do not need to solve the problems you write. For example, from the situation which follows: If the company buys one airplane of each kind, how much will it cost? If you need more space to write problems, use the back of this page.

An airline company is considering the purchase of 3 types of jet passenger airplanes, the 747, the 707 and the DC-10. The cost of each 747 is $15 million; $10 million for each DC 10; and $6 million for each 707. The company can spend a total of $250 million. After expenses, the profits for the company are expected to be $800,000 for each 747,$500,000 for each DC-10, and $350,000 for each 707. It is predicted that there will be enough trained pilots to man 25 new airplanes. The overhaul base for the airplanes can handle 45 of the 707 jets. In terms of their use of the maintenance facility, each DC 10 is equivalent to 1 1/3 of the 707’s and each 747 is equivalent to 1 2/3 of the 707’s.

1._________________________________________________________________________
2.___________________________________________________________________________
3.___________________________________________________________________________
4.___________________________________________________________________________
5.___________________________________________________________________________
6.___________________________________________________________________________
7.___________________________________________________________________________
8.___________________________________________________________________________
9.___________________________________________________________________________
10.__________________________________________________________________________
11.__________________________________________________________________________
12.__________________________________________________________________________
13.__________________________________________________________________________
14.__________________________________________________________________________
15.__________________________________________________________________________
APPENDIX C

GENERAL ASSESSMENT CRITERIA

The general assessment criteria can be applied to most problem posing scenarios. It possesses three criterions: *quantity*, *originality*, and *complexity*. Clearly, *quantity*, as an assessment criterion to problem posing activities, is exactly what it says – the number of generated responses to problem prompt. *Originality* is a relative criterion, but all the same, it is unique or less frequently seen responses. *Complexity* is, “the sophistication of the mathematical relationship embedded in the problems” (Silver & Cai, 2005).

The following four problem (Silver, 1997; Silver & Cai, 2005) could be scored using the General Assessment Criteria:

1. The radius of a circle inscribed in a square is 6 inches. Find out all that you can about the square and circle.

2. Pose problems that all can be solved using the **same division statement**: $540 \div 40 = ?$ How many different problems can you pose and solve? Two problems will be different only if they have **different answers**.

3. Ann has 34 marbles, Billy has 27 marbles, and Chris has 23 marbles. Write and solve as many problems as you can that use this information.

4. Write (at least) three different questions that can be answered from the information below. Jerome, Elliot, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliot. Elliot drove twice as many miles as Jerome. Jerome drove 50 miles.
APPENDIX D

ABBREVATED MATH ANXIETY SCALE (AMAS)

For each item, select one response that best represents how you feel in such a situation. Select one of the values between 1 (low anxiety) to 5 (high anxiety).

<table>
<thead>
<tr>
<th>Item</th>
<th>Low anxiety</th>
<th>Some what low anxiety</th>
<th>Medium anxiety</th>
<th>Somewhat high anxiety</th>
<th>High anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Having to use the tables in the back of a math book.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Thinking about an upcoming math test one day before.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Watching a teacher work an algebraic equation on the blackboard.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Taking an examination in a math course.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Being given a homework assignment of many difficult problems that is due the next class meeting.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Listening to a lecture in math class.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Listening to another student explain a math formula.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Being given a &quot;pop&quot; quiz in math class.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Starting a new chapter in a math book.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
### APPENDIX E

**SCORING RUBRIC TO ENCOURAGE DEPTH AND CREATIVITY**

<table>
<thead>
<tr>
<th>Assessment Criteria</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of Understanding</td>
<td>Little or no understanding.</td>
<td>Partial understanding; minor mathematical errors.</td>
<td>Good understanding; mathematically correct.</td>
<td>In-depth understanding; well-developed ideas</td>
</tr>
<tr>
<td>Fluency</td>
<td>One incomplete or unworkable approach.</td>
<td>At least one appropriate approach or related question.</td>
<td>At least two appropriate approaches or good related questions.</td>
<td>Several appropriate approaches or new related questions.</td>
</tr>
<tr>
<td>Flexibility</td>
<td>All approaches use the same method. (e.g., all graphs, all algebraic equations, and so on.)</td>
<td>At least two methods of solution (e.g., geometric, graphical, algebraic, physical modeling.)</td>
<td>Several methods of solution (e.g., geometric, graphical, algebraic, physical modeling.)</td>
<td></td>
</tr>
<tr>
<td>Originality</td>
<td>Method may be different but does not lead to a solution</td>
<td>Method will lead to a solution but is fairly common.</td>
<td>Unusual, workable, method used by only a few students.</td>
<td>Unique, insightful method used only by one or two students.</td>
</tr>
<tr>
<td>Elaboration or Elegance</td>
<td>Little or no appropriate explanation given.</td>
<td>Explanation is understandable but may be unclear in some places.</td>
<td>Clear explanation using correct mathematical terms.</td>
<td>Clear, concise, precise explanations making good use of graphs, charts, models, or equations.</td>
</tr>
<tr>
<td>Generalizations and Reasoning</td>
<td>No generalizations and Reasoning.</td>
<td>At least one correct generalization made; may not be well supported with clear reasoning.</td>
<td>At least one well-made, supported generalization, or more than one correct but unsupported generalization.</td>
<td>Several Well-supported generalizations; clear reasoning.</td>
</tr>
<tr>
<td>Extensions</td>
<td>None included, or extensions are not mathematical.</td>
<td>At least one related mathematical question appropriately explored.</td>
<td>One related question explored in depth, or more than one appropriately explored.</td>
<td>More than one related question explored in depth.</td>
</tr>
</tbody>
</table>
APPENDIX F

CRITERIA FOR CONVERGENT AND DIVERGENT THINKING

Balka (1974b) measured creative ability in mathematics using two major domains, namely convergent thinking and divergent thinking. He identified six criteria of which four were considered to be divergent and the other two were convergent.

1. The ability to formulate mathematical hypotheses concerning cause and effect in mathematical situation (divergent).
2. The ability to determine patterns in mathematical situations (convergent).
3. The ability to break form established mind sets to obtain solutions in a mathematical situation (convergent).
4. The ability to consider and evaluate unusual mathematical ideas, to think through their consequences for a mathematical situation (divergent).
5. The ability to sense what is missing from a given mathematical situation and to ask questions that will enable one to fill in the missing mathematical information (divergent).
6. The ability to split general mathematical problems into specific subproblems (divergent). (p. 634)
Office of the Vice President For Research
Human Subjects Committee
Tallahassee, Florida 32306-2742
(850) 644-8673 · FAX (850) 644-4392

RE-APPROVAL MEMORANDUM

Date: 10/12/2009

To: Vickie Lake [vlake@fsu.edu]

Address: 4459
Dept.: EDUCATION

From: Thomas L. Jacobson, Chair

Re: Re-approval of Use of Human subjects in Research
Contextual Problem Solving

Your request to continue the research project listed above involving human subjects has been approved by the Human Subjects Committee. If your project has not been completed by 10/11/2010, you must request renewed approval by the Committee.

If you submitted a proposed consent form with your renewal request, the approved stamped consent form is attached to this re-approval notice. Only the stamped version of the consent form may be used in recruiting of research subjects. You are reminded that any change in protocol for this project must be reviewed and approved by the Committee prior to implementation of the proposed change in the protocol. A protocol change/amendment form is required to be submitted for approval by the Committee. In addition, federal regulations require that the Principal Investigator promptly report in writing, any unanticipated problems or adverse events involving risks to research subjects or others.

By copy of this memorandum, the Chair of your department and/or your major professor are reminded of their responsibility for being informed concerning research projects involving human subjects in their department. They are advised to review the protocols as often as necessary to insure that the project is being conducted in compliance with our institution and with DHHS regulations.

Cc: Pamela Carroll, Chair [pcarroll@fsu.edu]
HSC No. 2009.3300

APPENDIX G
HUMAN SUBJECTS COMMITTEE APPROVAL LETTER
16 July 2008

Contextual Problem Solving

You are invited to be in a research study for the duration of this semester designed to explore preservice teachers' understandings and strategies of problem solving for contextual (informal or social) based problems and school (formal) based problems. You were selected as a possible participant because you are enrolled in one of the mathematics methods courses at FSU and FIU. We ask that you read this form and ask any questions you may have before agreeing to be in the study.

This study is being conducted by Dr. Vickie E. Lake, Mr. James Fetterly, and Dr. Ithel Jones in the School of Teacher Education of The Florida State University and Dr. Maria Fernandez at Florida International University.

Background Information:

Given that our purpose is to explore contextual problem solving strategies, the research questions guiding this study include: 1) examining evidence to support the degree of PST familiarity with the context and the number of solutions generated for the problem, 2) examining problem solving strategies used by preservice teachers in early childhood education, elementary education, and secondary education attending FSU and FIU, and 3) finding a correlation between contextual mathematics and science content and problem solving for early childhood education PST.

Procedures:

All procedures and assignment for this study are part of the regular course assignments. If you agree to be in this study, you are asked to do the following things:

1) Allow the researchers to copy your assignments and use the data for research purposes
2) Video-tape problem solving classroom sessions for later transcription
3) Complete content quizzes and pre and post surveys
4) Participate in focus group interviews

Risks and benefits of being in the Study:

Risks: Each student will be assigned a research number in order to protect his/her identify. The PI will be the only person to have record of this identifying information. There are no other known risk factors in their study.

Benefits: The students will benefit whether they participate or not by
1) an increase in problem solving knowledge,
2) an increase in creativity in problem solving solutions,
3) decrease in math (and science for ECE) anxiety by participating in courses that purposefully focus on contextually embedded knowledge.
Confidentiality:

Once participates are consented, they will be assigned a research number and all data will be coded and identified using that number. Only the PI will have access to original information and it will be stored on an external hard drive with password protection. The data will be kept in the PI's office on the FSU campus.

The records of this study will be kept private and confidential to the extent permitted by law. In any sort of report we might publish, we will not include any information that will make it possible to identify a participant. Research records and videotapes will be stored securely and only researchers will have access to the records. After five (5) years all data will be destroyed.

Voluntary Nature of the Study:

Participation in this study is voluntary. Your decision whether or not to participate will not affect your grade, current or future relations with either University. If you decide to participate, you are free to not answer any question or withdraw at any time without affecting these relationships.

Contacts and Questions:

The main researchers conducting this study are Vickie E. Lake, James Fetterly, and Maria Fernandez. You may ask any question you have now. If you have a question later, you are encouraged to contact at Florida State University or Florida International University, Lake: 850.644.1450; lake@coe.fsu.edu or Fetterly: 850.644.5458; fetterly@coe.fsu.edu or Fernandez: mfernan@fiu.edu.

If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher(s), you are encouraged to contact the FSU IRB at 2010 Levy Street, Research Building B, Suite 276, Tallahassee, FL 32306-2742, or 850-644-8633, or by email at jjcoper@fsu.edu.

You will be given a copy of this information to keep for your records.

Statement of Consent:

I have read the above information. I have asked questions and have received answers. I consent to participate in the study.

_________________________  _________________
Signature                                            Date
APPENDIX H

PERMISSION TO USE THE ABBREVIATED MATH ANXIETY SCALE

Subject: RE: AMAS
From: "Hopko, Derek R" <dhopko@utk.edu>
Date: Sunday, June 6, 2010 10:31 am
To: James Fetterly <jmf04m@fsu.edu>

Sure James, good luck with your research.

Here you go.
Derek

Derek R. Hopko, Ph.D.
Associate Professor and Associate Department Head
The University of Tennessee
Department of Psychology
307 Austin Peay Building
Knoxville, TN 37996-0900
PH: (865) 974-3368
FAX: (865) 974-3330

-----Original Message-----
From: James Fetterly [mailto:jmf04m@fsu.edu]
Sent: Sat 6/5/2010 7:54 PM
To: Hopko, Derek R
Subject: AMAS

Dear Professor Hopko,

As a doctoral candidate at Florida State University in mathematics education, I came across your instrument AMAS - Abbreviated Math Anxiety Scale. Currently, my research interests are in mathematical anxiety, beliefs, and creativity. I believe your instrument would prove very beneficial to my research. Would you allow me to utilize this tool in my study? Thank you for your time and consideration.

James Fetterly
LIST OF REFERENCES


BIOGRAPHICAL SKETCH

In 1975 James Fetterly was born in Adak, Alaska. He graduated high school in 1994 from Westside Christian School in El Dorado, Arkansas. At Central Baptist College in Conway, Arkansas, he earned an Associate in mathematics in 1996. Transferring to the University of Central Arkansas, in 2000 James obtained his Bachelor in mathematics with an emphasis in education. His master's degree, which he received in 2002, is also in mathematics education from the University of Central Arkansas. In Conway, Arkansas, he taught secondary mathematics at Conway High School East for five years and then at Carl Stuart Middle School he coached mathematics teachers for another two years. James Fetterly received his Specialist in mathematics education from Florida State University in 2007, and he has been a member of the Florida State University faculty since 2007 and is currently an associate in the School of Teacher Education. In previous semesters, he has taught mathematics methods courses for several programs. Although, his primary responsibility has been to teach elementary mathematics methods, James also has taught courses for middle and secondary mathematics education majors and special education majors. His research interests are in mathematical creativity.