THE FLORIDA STATE UNIVERSITY
COLLEGE OF EDUCATION

A COMPARISON OF VERBAL AND NONVERBAL INSTRUCTION
IN ELEMENTARY SCHOOL MATHEMATICS

By
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A Dissertation submitted to the Department of Mathematics Education in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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Approved:

[Signatures]

Professor directing Dissertation

Dean, College of Education
A COMPARISON OF VERBAL AND NONVERBAL INSTRUCTION IN ELEMENTARY SCHOOL MATHEMATICS

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Major Professor: Eugene D. Nichols

This study assessed the relative effectiveness of verbal and nonverbal teaching methods in facilitating the learning of mathematics. The two treatments differed only in that nonverbal instruction did not permit oral communication or use of written words.

Chalkboard instruction was characterized by complete silence in nonverbal classes. Instead of "calling on" a pupil, the teacher extended the chalk toward him and he went to the board to fill in the blank. A correct answer won a smile of approval, while an incorrect one was followed by the teacher's quizzical expression, then an extension of the eraser for correction by that student, or another if necessary.

In verbal classes, new terminology was introduced by writing the terms on the board and using them throughout the lesson. Students were lead to verbalize all generalizations developed in the lesson.

Instructional material consisted of four independent topics selected from different areas of mathematics: special geometric figures, lattice operations, ordered pair coordinates, and function machines. Daily lessons were taught on consecutive school days.

Teachers precisely followed the prescribed lessons at the chalkboard. Afterwards, identical worksheets for in-class practice were
distributed to both treatment groups.

Four fourth-grade classes consisting of 88 students in one school were randomly assigned to treatment groups so that two were taught nonverbally, and two by the conventional verbal method. Two teachers were assigned one class of each type. Treatment and teacher factors were crossed in a pretest-posttest control group design.

Analysis of covariance was performed using posttest scores as the dependent variable with pretest scores as a covariate. There was no significant difference between treatments or teachers, nor any interaction between them. Also, there was no interaction between treatment and level (high or low) of either aptitude or achievement, measured by California Test scores (Comprehensive Tests of Basic Skills — Level 1 Form Q and Short Form Test of Academic Aptitude — Level 2).

The demonstrated comparability of the two teaching methods not only points to nonverbal instruction as an alternate mode, but also seriously questions the effectiveness of conventional teacher talk in enhancing learning.

Teachers with a creative bent should be encouraged to experiment with nonverbal instruction and design activities for all levels of development. The technique could be used effectively to break the routine of conventional instruction.

The importance of nonverbal components should be stressed in methods courses for pre- and in-service teachers. Techniques of nonverbal instruction should be practiced in student teaching practicums.
ACKNOWLEDGEMENTS

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The author wishes to acknowledge Lillian Ruediger School of Tallahassee, Florida as the site of the study. In particular, Principal Mr. San Alderman and Curriculum Coordinator Mrs. Ethel Ruff for coordinating the research effort. Also fourth-grade teachers Mrs. Barbara Brown, Mrs. Brenda Morrison, Mrs. Eliza Russ, and Mrs. Charlotte White for cooperating in providing their students as subjects for the experiment.

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CHAPTER I

STATEMENT OF THE PROBLEM

The purpose of this study was to compare two modes of instruction, verbal and nonverbal, in elementary school mathematics.

Description of Nonverbal Instruction

Hedges (1967) captures the essence of nonverbal teaching when he speculates "I have often thought how wonderful it would be if teachers were forced to teach one entire week of each year with their mouths taped shut [p. 335]." Although it may be debated whether or not tape would be necessary (the investigator wonders how long most would last without it), there is general agreement that the essential feature of nonverbal instruction is the total absence of oral communication and use of written words.

Halpin (1970) defines nonverbal communication, or muted language as he calls it, as the language of eyes and hands, of gesture, etc., whereas its verbal counterpart is characterized by any form of spoken communication or written word.

A pioneer at teaching without talking, "irtz (1963) offers this example of the technique:

The first-grade teacher puts the sketches shown in Figure 1 on the board, maintaining complete silence.

\[ \text{Figure 1} \]
The teacher stops. Looks quizzically at the board for a moment. She
moves toward the board as if to continue, but changes her mind
and looks at the class.
Hands go up. She extends the chalk toward a child in the back of
the room. Without a sound from the group, he comes up and goes to
work on any of the sketches. His success wins the smile of approval
and other volunteers continue the activity [p.72].

Review of Literature

Verbal activity in the classroom has been the object of much re-
search in education. According to Flanders (1965), "The chances are
better than 60 percent that you will hear someone talking if you are in
an elementary or secondary school classroom [p.1]." Keux and Smith
(Biddle & Ellena, 1964) write "Teaching behavior is primarily verbal
[p.129]."

Studies focus variously on the amount of classroom talk, its source
(teacher or pupil), and how it may be classified. Flanders (1965) found
that teachers tend to do most of the talking — about 70% in the aver-
age classroom. Bellack (Cross & Kelle, 1970) found them talking approx-
imately three times more than students, and that this imbalance is a
relatively stable characteristic in almost any classroom.

Floyd (1963) conducted a large-scale investigation using a sample
of 40 elementary schools selected randomly from 253. In each school,
the principal recommended his best primary teacher, whose class was used
for the study. Data consisted of one-hour tape-recordings from 30 of
the classes, together with observer-recordings of an entire day in the
remaining ten.

The total word-count comparison from the tapes revealed a teacher-
pupil ratio of 21:29. The less than three-tenths words spoken by stu-
dents were divided among them, yielding a comparatively infinitesimal
amount for an individual student.
The average teacher-pupil question-asking ratio was 96:4 for taped classes, and 95:5 for observed classes. Classification of teachers' questions revealed that about 85% fell into the categories of memory, information, requests or direction-giving, criticism or evaluation, and comparison, the majority belonging in the first two categories. Pupil questions largely reflected uncertainty, were of a checking nature ("What did you say?"), or were requests for permission to speak. They showed a minimum of real interest or serious thought.

Floyd concluded that:

1. These teachers dominated the oral activity, and were doing most of the work themselves.
2. By their methods of asking questions, the teachers encouraged guessing and slovenly habits of thought.
3. So many of the questions required only the use of memory to answer them, it indicates that memorization is of major significance, and could be the main goal of instruction.
4. Teachers were acting as cross examiners, relatively uninterested in the needs, interests, and capacities of their pupils.
5. These teachers seemed unaware of the psychology of oral questioning. Not only were they unskilled in the use of questions as an instructional instrument, they did not even seem to realize such a skill could be developed [p.150].

Similar findings are reported by Minskoff (1967) with a sample of 17 teachers of special classes for the mentally retarded. Analysis of tape-recordings of a full day for each class revealed that practically all of the teachers' questions were in the cognitive-memory category, with very few in the productive thinking classes. They were preoccupied with management and routine problems.

Fey (1962) studied patterns of verbal communication from tape-recordings of four sessions each of five different classes participating in the Secondary School Mathematics Curriculum Improvement Study at Columbia University. His unit of analysis, a "move", is defined by Bellack (Fey, 1970) as an uninterrupted utterance or partial utterance of a
single speaker which serves the pedagogical purpose of structuring the
discourse, soliciting information or action, responding to a solicita-
tion, or reacting to a prior move. Fey found that:

1. Each teacher spoke more moves and more lines (of tape-script) than all of his students. The ratio was 3 to 2 in terms of
   moves and 3 to 1 in terms of lines.
2. Each teacher dominated the pedagogical functions of structuring (80 percent of these moves), soliciting (95 percent of
   these moves), and reacting (85 percent of these moves), leaving
   responding as the major student activity.
3. Over 50 percent of all moves were statements or questions of
   fact, 25 percent evaluations (mostly by teachers), and the re-
   maining moves divided between justifying and analytic process
   [p.3040A].

Since teacher talk dominates classroom activity, its instructional
effectiveness has been investigated by numerous researchers. Corrigan
(1968) collected data from the classes of 25 college professors at
teacher-preparing institutions in Pennsylvania. Teacher verbal behavior
was correlated with student perceptions of the teacher, as measured by
the Witiol Climate Index for categorizing teachers' verbal behavior and
a student perception scale constructed by the investigator. Little or
no significant relationship was found for the following groups: all
students, males, elementary education students, secondary education stu-
dents, and liberal arts students. Only female student perception of the
teacher correlated significantly with teacher verbal behavior.

For 11 sixth-grade classes in New York, Lamanna (1968) found that
teacher verbal behavior patterns affect achievement in mathematics com-
putation for average and above-average students, but not necessarily
achievement in problem solving and concepts.

As a result of his study on how children develop skills for inquiry
and problem solving, Bruner (Cross & Nagle, 1970) concluded that "The
more one has practice, the more one is likely to generalize what one has
learned into a style of problem solving or inquiry that serves for any kind of task encountered... [p.22]." Other research indicates that students who interact with what they are learning probably learn more effectively and efficiently than those who do not.

Hedges (1967) writes "When the teacher is doing most of the talking, the classroom is teacher centered, teacher directed, teacher dominated. The teacher, in short, is doing the explaining, setting the goals, and performing the evaluation. The teacher is at the center of the stage and hence is doing most of the learning. He is doing most of the learning because he is the one who is the most involved in the whole process. Learning theory tells us that we learn most effectively when we are involved, when we participate actively... [p.335]."

Cross and Hazel (1970) point out "Quite clearly, then, when a teacher dominates the classroom discussion he is actually inhibiting the involvement of his students in the learning process, stunting their imagination and creativity, and rapidly encouraging them to tune him out. The end result is reduced student learning... [p.22]."

A case in point is offered by Holt (1967):

Once I was teaching a fifth-grade math class and was very much pleased with myself because, instead of "telling" a youngster, I was "making her think" by asking questions. But she didn't answer. I followed each question with another that was easier and more pointed. Still no answer. I looked hard at my silent student and discovered she didn't even look puzzled. Just patient. Then it dawned on me: She was just waiting for that really pointed question — the one that would give her the right answer.

When a child gets answers by such means, it does double harm: He doesn't learn and he comes to believe that a combination of bluffing, guessing, mind reading, snatching at clues, and getting answers from other people is what school is all about...

One result of too much teacher talk is that children who, when they were little, were turned on all the time, learn to tune themselves off or down. They listen with only a small part of their being, like any adult listening to a boring talk. They lose the knack of it and the taste for it. It is a great loss... [pp.17-18].
In summary, Hedges (1967) states "Most teaching, from first grade through graduate school, can be described in a single sentence: "Teaching is talking." This cannot, however, be followed with its logical corollary: "Learning is listening" [p.33]."

If it is not the case that "Teaching is talking," then what other means can teachers resort to? Bensen (1968) replies "It should be emphasized that a great deal of interaction can take place between the teacher and the learner without the utilization of verbal behavior. This communication [nonverbal] ... can be very effective if utilized appropriately [p.31]."

Garner (1970) contends that "This type of communication often is overlooked not only by our teachers, but also by our methods courses for future teachers and student teaching practicums. The use of verbal and nonverbal communication should be made known to future teachers ... [p.364]."

After analyzing video tapes of five class sessions of eight biology classes (four BSCS and four non-BSCS), Balzer (1968) concluded that "Nonverbal behaviors may constitute a much greater proportion of the total behaviors of the teacher than has been hitherto reported. Nonverbal components influencing the teaching-learning situation were found to occur in 65.22 per cent of all behaviors of the teachers studied [p.1353]."

Unfortunately, studies of nonverbal teacher behavior are both limited and macroscopic (Grant, 1970). Wirtz (1963) calls for further research in nonverbal instruction and states his case for its place in education:

Perhaps one key to the success of "nonverbal instruction" is that we can't call them [variables] anything ---- we don't utter a word.
We avoid the controversy. . .
The attention is focused on the pattern — not on words or names for anything. . .

Later when we do resort to verbalization, we can rest assured that any questionable formulation will be less confusing to children because they had already grasped the idea firmly and clearly before we tried to talk about it.

Perhaps light will be shed on the encouraging results we have obtained as they are analyzed as extensions of the important pioneering work of Gertrude Hendrix at the University of Illinois in "Learning through discovery" and the important role played by "nonverbal awareness".

Completely "nonverbal instruction" literally forces us to invent ways to lead children toward making independent discoveries. Further, the rules of the game require an extended opportunity for children to use their discoveries before they hear any word even remotely attached to them. They are guaranteed an extended period of "nonverbal awareness" and repeated opportunities during that interim to test and revise and demonstrate whatever it is they have discovered.

What is the mechanism that leads children to impose utter silence on themselves for an extended period of time? Why is the level of motivation of all children so high when instruction is of the "nonverbal" kind . . . ? How often should this technique be employed for what period of time? What kind of verbal references, if any, should be made in retrospect after such an experience?

These are unanswered questions that need exploration by competent investigators.

Teachers with a creative bent ought to be encouraged to experiment carefully with "nonverbal" instruction and design their own activities at different levels of development. Let us all accept a challenge: Let's find simple ways and means that could be used to lead children to discover every fact and algorithm of arithmetic — including the division of one fraction by another — through "nonverbal" activities. To meet this challenge would certainly not culminate in a nonverbal curriculum in arithmetic. It would, however, provide a wealth of activities that could be drawn upon judiciously to deepen mathematical understanding and stimulate mathematical creativity [pp. 66-77].

The Present Study

At the Cortisol annual meeting of the National Council of Teachers of Mathematics in 1962, Wirtz called for research in nonverbal instruction. Ten years later, there is still no report of any experimental investigation of the method.

Yet studies continue to report overwhelming proportions of teacher talk, with the comparatively minute percentage of student involvement
diverted from productive thinking. The instructional effectiveness of most teacher verbal behavior has been seriously questioned. In spite of evidence of the important role of nonverbal components, little research has been devoted to this area.

It was the purpose of the present study to determine whether or not nonverbal teaching can be used effectively as a mode of instruction. The nonverbal method was compared with verbal teaching of the same subject matter, the two treatments differing only in that the latter was accompanied by oral communication and use of written words.

The dependent variable in this study (i.e., measure of instructional effectiveness of the treatments) was student learning as measured by adjusted posttest scores, using pretest scores as a covariate. (This treatment is recommended by Campbell and Stanley (1963) in preference to use of simple gain scores.)

If nonverbal instruction proved as effective as its verbal counterpart, the result would be not only to open the door to use of this unconventional technique, as Wirtz encourages, but also to seriously question the effectiveness of conventional teacher talk in enhancing learning.
CHAPTER II

DEVELOPMENT OF MATERIALS

Preliminary Materials

Subject matter for instruction consisted of four independent topics appropriate for single-lesson development, selected from different areas of mathematics:

(a) special geometric figures
(b) lattice operations
(c) ordered pair coordinates
(d) function machines.

The lessons in their final form are given in Appendix A, along with corresponding worksheets, the pretest and posttest, and teacher guidelines for the research materials.

The sampling of several topics was preferred to an in-depth development of one topic, for example, in order to avoid a possible bias towards content that might lend itself better to one method of instruction than to the other. Also, it was desired to demonstrate the applicability of nonverbal instruction to diverse areas of mathematics.

Content was chosen to be appropriate for, but unfamiliar to, students in the population sampled from. A minimum of prerequisite skills was necessary — only the function machine lesson required some computational skill.

Specifically the lessons were four in number so that each day of instruction (Monday - Thursday) and the day of the posttest (Friday) would not be separated by any days school was not in session. The
pretest could then be administered the previous Friday, with some lapse of time to minimize its effect.

Pilot Study

For purposes of developing the experimental materials, it was decided to perform a pilot study in March, 1973. Subjects for the trial run were fourth-graders at the Developmental Research School of Florida State University, Tallahassee, Florida.

As there were only two classes at the grade-level, it was decided that the most benefit would be derived by having each teacher instruct one class nonverbally in all four lessons. Although both teachers, Dr. Eugene Nichols and Dr. Herbert Wills of the Department of Mathematics Education of Florida State University, were experienced in nonverbal instruction, it had been a while since either practiced the technique. The pilot study, then, allowed for equal practice for both teachers with each lesson, and for the subsequent revision of each.

Thus, the trial run was conducted exactly as the actual experiment was planned, except for the omission of a control group (verbal treatment). The pre-experimental design employed:

<table>
<thead>
<tr>
<th>Treatment I</th>
<th>Class I</th>
<th>Class II</th>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Teacher II</td>
<td></td>
<td></td>
</tr>
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</table>

is described by Campbell and Stanley (1963) as the one-group pretest-posttest design.

Revision of Materials

Based on the pilot study, final revisions of the experimental materials were made. These are given in Appendix A.

All lessons were reduced in length, but only slightly modified in
format. No major content changes were made except in the geometry lesson. Since its required student responses had a 50% chance of being correct, this lesson was placed initially.

Although each lesson was completely independent of the others, it was expected that there might be some transfer between the lattice operations and ordered pair coordinates lessons, since both use rectangular arrays as a frame of reference. Therefore, these lessons were placed consecutively.

The function machine lesson, which proved most difficult in the pilot study and required computational proficiency, was placed terminally. It was also expected that there might be some transfer from the lattice operations lesson since both develop the notions of composition of operations and inverses.

The ordering of lessons then was as follows:

Lesson 31 — special geometric figures
Lesson 32 — lattice operations
Lesson 33 — ordered pair coordinates
Lesson 34 — function machines.

To increase student interest and work efficiency on accompanying worksheets, it was decided to require that each page be checked and corrected before going on to the next. The experimenter would assist each teacher in performing this function of immediate feedback and reward. The number of questions per page on the worksheets was adjusted accordingly. (Only worksheet 32 required considerably smaller segmentation so that half-size pages were used for it.)

Pretest and posttest length was not altered. The only changes in content and format corresponded to revisions of the original lessons. As it was expected that pretest performance might be higher on items from lessons 31 and 34, these test questions were placed initially.
Since the purpose of the pilot study was to develop the research materials rather than to investigate trends in results, no statistical analysis of the data collected was made.
CHAPTER III

DESCRIPTION OF STUDY

Subjects

Subjects for the experiment were fourth-graders at Lillian Ruediger School in Tallahassee, Florida. The school was selected upon satisfying the following criteria:

(a) minimum of four classes at the grade level
(b) self-contained classrooms
(c) random assignment of pupils to classes.

The fourth-grade level was chosen based on its suitability for the difficulty of the subject matter and the prerequisite skills required.

Although 106 students in four classes participated in the study, 12 were absent from testing and six more missed some of the instruction, leaving a total of 89 participants present for pretest, posttest, and complete treatment. There was no differential loss of subjects due to absence during treatment across either methods or teachers.

Complete California Test scores (Comprehensive Tests of Basic Skills — Level 1 Form Q and Short Form Test of Academic Aptitude — Level 2) are presented in Appendix E, for 74 of these students for which the data was available. (The battery was administered in April, 1972.) The national mean for aptitude measures is 100, whereas for grade-equivalent achievement scores, it is 37 (third year, seventh month). Means for each class and the overall group are given in Table 1.
TABLE 1

TABLE 1
EXPERIMENT ON CALIFORNIA TEST SCORES

<table>
<thead>
<tr>
<th>Class</th>
<th>Language TQ</th>
<th>Nonlanguage TQ</th>
<th>Read Achievement</th>
<th>Language Achievement</th>
<th>Arithmetic Achievement</th>
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<tbody>
<tr>
<td>I</td>
<td>0.95</td>
<td>0.96</td>
<td>25</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>II</td>
<td>0.99</td>
<td>1.00</td>
<td>37</td>
<td>40</td>
<td>37</td>
</tr>
<tr>
<td>III</td>
<td>0.92</td>
<td>0.93</td>
<td>32</td>
<td>34</td>
<td>26</td>
</tr>
<tr>
<td>IV</td>
<td>1.03</td>
<td>1.11</td>
<td>42</td>
<td>48</td>
<td>43</td>
</tr>
<tr>
<td>Overall</td>
<td>0.96</td>
<td>1.01</td>
<td>36</td>
<td>39</td>
<td>38</td>
</tr>
</tbody>
</table>

Procedure

The experiment was conducted April 27 and April 30 - May 4, 1973, in the fourth-grade classrooms of Lillian Huediger School. Although regular classes were kept intact, their normal teachers were not present during the study.

The pretest and posttest (alternate forms) were administered by the experimenter on consecutive Fridays (April 27 and May 4) in the morning. Time allotted for the testing periods was 20 minutes.

In the interim (April 30 - May 3), instruction was given by Dr. Eugene Nichols and Dr. Herbert Wills of the Mathematics Education Department of Florida State University. Daily one-hour periods of instruction for the four classes were scheduled at 8:35 and 9:40 for one teacher, and at 11:35 and 1:30 for the other.

The nature of the instruction was as described in the teacher guidelines of Appendix A. Teachers exactly duplicated at the chalkboard the presentation following the guidelines for each lesson. The only variation between presentations given the verbal and nonverbal classes was the additional use of oral communication and written words in the
Verbal classes.

Nonverbal classes, on the other hand, were characterized by complete silence. Instead of "calling on" a pupil, the teacher extended the chalk toward him and he went to the board to fill in the blank. (Whenever another type of response was required — circling the correct answer, locating and labelling a point, or drawing an arrow to indicate which group a figure belongs with — the teacher provided an example to cue students to the procedure.) A correct answer won a smile of approval, while an incorrect one was followed by the teacher's quizzical expression, then an extension of the eraser for correction.

In the verbal classes, new terminology was introduced by first writing the terms on the chalkboard, then using them throughout the lesson. The teacher lead students to verbalize all generalizations developed at the board. The teacher was allowed to pursue any point brought up by a student, and to supplement the lesson with further oral explanation.

The experimenter served as a trained observer, making note of significant differences between the two groups. In particular, a tally was kept of all student errors at the chalkboard, for comparison. Verbal classes were taped and their transcription is given in Appendix C. No transcription is given for two sessions which did not tape.

After each lesson, identical worksheets were distributed to both groups for independent in-class practice with the material covered at the board. Students were required to have each page checked and corrected before going on to the next. The experimenter assisted each teacher in the grading task.

The entire experiment was carried out with no deviation from the
planned format.

Experimental Design

The four intact classes were randomly assigned to treatment groups so that two were taught by the nonverbal method (Treatment I), and two by the conventional verbal method (Treatment II). Scheduling determined the further assignment of teachers to classes so that each instructor taught one class by the nonverbal method, and one class verbally. To avoid a possible bias due to the order of treatment presentation, one teacher taught nonverbally first, then by the verbal method, and the other, vice-versa.

Hence, the following cross-factor design:

<table>
<thead>
<tr>
<th>Teacher I</th>
<th>Class I</th>
<th>Class III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher II</td>
<td>Class II</td>
<td>Class IV</td>
</tr>
</tbody>
</table>

described by Campbell and Stanley (1963) as the pretest-posttest control group design. It should be noted, however, that this design is quasi-experimental in the sense that intact comparison groups were used instead of randomly assigning subjects to treatments.

Statement of Hypotheses

Specific hypotheses tested were:

1. Main effects
   
   (a) There is no difference between means of adjusted posttest scores attributable to treatment.
   
   (b) There is no difference between means of adjusted posttest scores attributable to teachers.

2. Interaction effects
   
   (a) There is no interaction between treatment and teachers with respect to adjusted posttest scores.
(b) There is no interaction between treatment and language IQ with respect to adjusted posttest scores.

(c) There is no interaction between treatment and nonlanguage IQ with respect to adjusted posttest scores.

(d) There is no interaction between treatment and reading achievement with respect to adjusted posttest scores.

(f) There is no interaction between treatment and language achievement with respect to adjusted posttest scores.

(g) There is no interaction between treatment and arithmetic achievement with respect to adjusted posttest scores.
CHAPTER IV

ANALYSIS OF RESULTS

Results

Raw data collected from the experiment is given in Appendix B for 83 subjects present for pretest, posttest, and complete treatment. It may be noted that all but two subjects gained from pretest to posttest, and losses were only one and two points.

Means for each class and the overall group are given in Table 2.

| TABLE 2 |
| MEANS OF PRETEST AND POSTTEST SCORES |
|-----------------|-----------------|-----------------|-----------------|
| Treatment | Teacher | Pretest | Posttest |
| 1 | 1 | 17.7 | 71.9 |
| 1 | 2 | 16.9 | 71.4 |
| 2 | 1 | 19.4 | 66.0 |
| 2 | 2 | 27.0 | 80.5 |
| Overall | | 20.7 | 72.9 |

It should be remarked that chance score (resulting from guessing at items with a 50% chance of being correct) expected value is 19.0 on both alternate forms.

A tally of student errors made at the chalkboard in each class is presented in Table 3.
TABLE 3
TALLY OF STUDENT ERRORS AT CHALKBOARD

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Teacher</th>
<th>Number of Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>19</td>
</tr>
</tbody>
</table>

Statistical Test Employed

As recommended by Campbell and Stanley (1963), analysis of covariance was performed using posttest scores as the dependent variable with pretest scores as a covariate. Although the authors caution that formulas provide too small an error term when intact classes are employed, in no case were resulting F ratios large enough to reject the stated hypotheses anyway. Also, there was no apparent violation of any underlying assumptions of analysis of covariance.

Since the employment of pretest scores as a covariate presumes a significant correlation with posttest scores, its value was computed, along with correlations between all other measures. These are given in Table 4 for the overall group.

Although the correlation between pretest and posttest scores is moderate, it may be considered significant for experiments in education. The highest correlations obtained were between pairs of California Test scores, and between California Test and posttest scores, and between posttest and gain scores. Moderate correlations were found between California Test and pretest scores, as well as between California Test and gain scores.
TABLE 4

CORRELATIONS BETWEEN SCORES

<table>
<thead>
<tr>
<th>Scores</th>
<th>Lang IQ</th>
<th>Nonlang IQ</th>
<th>Read Achievmt</th>
<th>Lang Achievmt</th>
<th>Arith Achievmt</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lang IQ</td>
<td>1.00</td>
<td>.69</td>
<td>.86</td>
<td>.84</td>
<td>.61</td>
<td>.40</td>
<td>.69</td>
<td>.52</td>
</tr>
<tr>
<td>Nonlang IQ</td>
<td>.69</td>
<td>1.00</td>
<td>.74</td>
<td>.77</td>
<td>.71</td>
<td>.48</td>
<td>.73</td>
<td>.51</td>
</tr>
<tr>
<td>Read Achievmt</td>
<td>.86</td>
<td>.74</td>
<td>1.00</td>
<td>.90</td>
<td>.81</td>
<td>.49</td>
<td>.65</td>
<td>.42</td>
</tr>
<tr>
<td>Lang Achievmt</td>
<td>.84</td>
<td>.77</td>
<td>.90</td>
<td>1.00</td>
<td>.80</td>
<td>.56</td>
<td>.71</td>
<td>.44</td>
</tr>
<tr>
<td>Arith Achievmt</td>
<td>.81</td>
<td>.71</td>
<td>.80</td>
<td>1.00</td>
<td>.39</td>
<td>.69</td>
<td>.53</td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>.40</td>
<td>.48</td>
<td>.49</td>
<td>.56</td>
<td>.39</td>
<td>1.00</td>
<td>.42</td>
<td>-.16</td>
</tr>
<tr>
<td>Posttest</td>
<td>.69</td>
<td>.73</td>
<td>.65</td>
<td>.71</td>
<td>.69</td>
<td>.42</td>
<td>1.00</td>
<td>.83</td>
</tr>
<tr>
<td>Gain</td>
<td>.52</td>
<td>.51</td>
<td>.42</td>
<td>.44</td>
<td>.53</td>
<td>-.16</td>
<td>.83</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Analyses

Results of the analysis of covariance obtained from partitioning by treatment or teaching method (T) and teachers (T) are given in Table 5.

TABLE 5

ANALYSIS OF COVARIANCE BY TREATMENT AND TEACHER

<table>
<thead>
<tr>
<th>SV</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>134.04</td>
<td>134.04</td>
<td>.30</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
<td>446.93</td>
<td>446.93</td>
<td>1.00</td>
</tr>
<tr>
<td>MT</td>
<td>1</td>
<td>436.96</td>
<td>436.96</td>
<td>.97</td>
</tr>
<tr>
<td>S/T</td>
<td>83</td>
<td>37221.76</td>
<td>442.45</td>
<td></td>
</tr>
</tbody>
</table>

As none of the obtained F ratios exceeds the critical value \( F_{.05}(1, 83) \approx 3.97 \), it is not possible to reject any of the first three
null hypotheses stated. That is, there was no significant treatment or

teacher main effect, nor a significant interaction between the two.

There was no significant difference in adjusted posttest scores due to
either treatment or teachers, nor did the effects of one factor change
markedly over levels of the other.

To test for interactions between treatment and each California Test
measure, high and low levels of these scores were specified. High
scores were designated at or above 100 (national mean) for aptitude
measures, and at or above 37 (grade equivalent) for achievement meas-
ures, while low scores fell below these critical values.

Results of the analysis of covariance obtained by partitioning by
treatment and level for each California Test measure (Language IQ (V),
Nonlanguage IQ (N), Reading Achievement (R), Language Achievement (L),
and Arithmetic Achievement (A)) are presented in Tables 6 - 10.

As none of the obtained F ratios for interaction exceeds the criti-
cal value $F_{05}(1, 69) \approx 3.99$, it is not possible to reject any of the
last five null hypotheses stated. There was no significant interaction
between treatment and any one of the California Test measures.

\begin{table}[h]
\centering
\caption{Analysis of Covariance by Treatment and V-Level}
\begin{tabular}{|c|c|c|c|c|}
\hline
SV & df & SS & MS & F \\
\hline
K & 1 & 176.35 & 176.35 & .57 \\
V & 1 & 3574.75 & 3574.75 & 11.54* \\
L & 1 & 354.31 & 354.31 & 1.14 \\
S/LV & 69 & 21373.57 & 309.76 & p < .005 \\
\hline
\end{tabular}
\end{table}
### Table 7

**Analysis of Covariance by Treatment and D-Level**

<table>
<thead>
<tr>
<th>SV</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>18.67</td>
<td>18.67</td>
<td>.07</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>7092.13</td>
<td>7092.13</td>
<td>26.51*</td>
</tr>
<tr>
<td>MR</td>
<td>1</td>
<td>132.33</td>
<td>132.33</td>
<td>.49</td>
</tr>
<tr>
<td>S/HR</td>
<td>69</td>
<td>18460.57</td>
<td>267.54</td>
<td>*p &lt; .001</td>
</tr>
</tbody>
</table>

### Table 8

**Analysis of Covariance by Treatment and R-Level**

<table>
<thead>
<tr>
<th>SV</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>113.92</td>
<td>113.92</td>
<td>.38</td>
</tr>
<tr>
<td>R</td>
<td>1</td>
<td>4390.79</td>
<td>4390.79</td>
<td>14.62*</td>
</tr>
<tr>
<td>MR</td>
<td>1</td>
<td>242.62</td>
<td>242.62</td>
<td>.81</td>
</tr>
<tr>
<td>S/HR</td>
<td>69</td>
<td>20727.62</td>
<td>300.40</td>
<td>*p &lt; .001</td>
</tr>
</tbody>
</table>

### Table 9

**Analysis of Covariance by Treatment and L-Level**

<table>
<thead>
<tr>
<th>SV</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>48.93</td>
<td>48.93</td>
<td>.17</td>
</tr>
<tr>
<td>L</td>
<td>1</td>
<td>5477.28</td>
<td>5477.28</td>
<td>18.81*</td>
</tr>
<tr>
<td>ML</td>
<td>1</td>
<td>78.50</td>
<td>78.50</td>
<td>.27</td>
</tr>
<tr>
<td>S/ML</td>
<td>69</td>
<td>20093.91</td>
<td>291.22</td>
<td>*p &lt; .001</td>
</tr>
</tbody>
</table>
TABLE 10
ANALYSIS OF COVARIANCE BY TREATMENT AND A-LEVEL

<table>
<thead>
<tr>
<th>SV</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>174.10</td>
<td>174.10</td>
<td>.80</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>10518.78</td>
<td>10518.78</td>
<td>48.50*</td>
</tr>
<tr>
<td>MA</td>
<td>1</td>
<td>29.63</td>
<td>29.63</td>
<td>.14</td>
</tr>
<tr>
<td>S/MA</td>
<td>69</td>
<td>14945.59</td>
<td>216.89</td>
<td>*p &lt; .001</td>
</tr>
</tbody>
</table>

Although no hypotheses were stated regarding California Test score effects, it may be noted that F ratios for these effects exceed the critical value $F_{.001}(1, 69) \approx 11.93$ in all but one case, for which the critical value must be lowered to $F_{.005}(1, 69) \approx 3.44$ in order to reject the null hypothesis. As would be expected, there was a significant difference in adjusted posttest scores across levels of aptitude and achievement — those with higher California Test scores had higher adjusted posttest scores.
Discussion of Results

In retrospect, it was the purpose of this study to compare verbal and nonverbal teaching methods, and to determine whether or not nonverbal teaching can be used effectively as a mode of instruction.

The comparison of teaching methods revealed no significant difference between them in terms of measured student learning. Nonverbal instruction proved as successful as its verbal counterpart, thus demonstrating its effective use as a teaching technique.

As to possible reasons for this outcome, one could not point to "unmatched" groups, since an analysis of variance revealed no significant difference between them in language aptitude and achievement, as well as reading achievement. In fact, verbal classes had significantly higher non-language aptitude and arithmetic achievement scores.

Some (e.g., Gertrude Hendrix) might hypothesize that requiring students to verbalize generalizations and introducing difficult terminology did not contribute to or may have actually interfered with learning in verbal classes. Thus no benefit was reaped from the additional verbal activity, so there was no difference in performance between treatment groups.

This finding supports Benson's (1962) contention that nonverbal communication can be very effective, and that such interaction can take
place between teacher and pupil without verbal activity. It also substantiates Balzer's (1968) conclusion that nonverbal components have a much greater influence on the teaching-learning situation than has been thus far reported.

In addition to no difference between treatment groups, it was also found that there was no differential effect across methods in terms of aptitude or achievement level. Nonverbal instruction was not markedly less effective than verbal teaching with high or low level students.

Recommendations

The demonstrated comparability of these teaching methods not only points to nonverbal instruction as an alternate mode, but also seriously questions the effectiveness of conventional teacher talk in enhancing learning.

Certainly a totally nonverbal curriculum is not advocated. However, particularly for a subject such as mathematics, teachers with a creative bent should be encouraged to experiment with nonverbal instruction and design activities for all levels of development, as Wirtz (1963) encourages. The technique might be used less frequently as a change of pace from conventional instruction.

Since, as Garner (1970) points out, nonverbal communication is overlooked by teachers and future teachers alike, the importance of these components should be stressed in methods courses for pre- and in-service teachers, and in student teaching practices.

Limitations and Suggestions for Further Research

The chief limitation of the present study arose from the small number of subjects involved. In addition to the primary objective of detecting any difference between treatment groups, the investigator also
sought to determine whether or not students with high nonverbal but low verbal aptitude and/or achievement did better via nonverbal instruction (or vice-versa). Unfortunately, the number of subjects in these cells was too small to base valid conclusions upon. (Generally, students who scored high in one area scored high in the other, and vice-versa.) A larger number of subjects might allow for a proper comparison.

In choosing lessons for experiments similar to this, researchers should try to select material requiring virtually no prerequisite skills. Many students encountered difficulty with the function machine lesson because of computational deficiency. Due to this interference factor, such material should be avoided.

The experimenter also suggests more exact specification of when new terminology is to be introduced and generalizations verbalized, than was given here. Teacher verbal behavior should be more uniformly controlled.

Other studies might isolate specific types of verbal activity for comparison. Several distinct verbal behaviors emerged in the present study — teacher introduced new terminology, teacher wrote new term on chalkboard, teacher used new term in lesson, student used new term in lesson, teacher verbalized generalization, student verbalized generalization, teacher restated student verbalization, teacher wrote generalization on chalkboard, etc. The separate and joint contributions of these behaviors should be investigated.

Classification systems should be devised for nonverbal components of instruction similar to those developed by Fay (1968) and others for verbal classroom behavior. Such systems would enable more precise analysis and comparison of the two aspects and their integration into total teacher behavior. Hopefully as a result, a more optimal balance may be
achieved between verbal and nonverbal components of the learning situation.

Such knowledge should be communicated to teachers and prospective teachers. The importance of nonverbal components should be stressed in methods courses, and techniques of nonverbal instruction should be practiced in student teaching practicums. Investigation should be made of resulting changes in these teachers' classroom behavior and subsequent effects on student learning.
APPENDIX A

EXPERIMENTAL MATERIALS
Teacher Guidelines
Prior to instruction, students will be administered a pretest (alternate form of the posttest) by the experimenter. Oral directions will be given, and students will be assured that they are not expected to answer the items correctly. Rather, the sole purpose of the test is to determine how much of the upcoming material students may have already mastered.

Instructional material consists of four independent topics appropriate for single-lesson development, selected from different areas of mathematics. Teacher guidelines precede each lesson.

Teachers will exactly duplicate at the chalkboard the presentation following the guidelines for each lesson. The only variation between presentations given the verbal and nonverbal classes will be the additional use of oral communication and written words in the verbal classes.

Nonverbal classes, on the other hand, will be characterized by complete silence. Instead of "calling on" a pupil, the teacher will extend the chalk toward him and he will go to the board to fill in the blank. (Whenever another type of response is required — circling the correct answer, locating and labelling a point, or drawing an arrow to indicate which group a figure belongs with — the teacher will provide an example to cue students to the procedure.) A correct answer wins a smile of approval, while an incorrect one is followed by the teacher's quizzical expression, then an extension of the eraser for correction.

In the verbal classes, new terminology will be introduced by first writing the terms on the chalkboard, then using them throughout the lesson. The teacher will lead students to verbalize all generalizations developed at the board. The teacher is allowed to pursue any point
brought up by a student, and to supplement the lesson with further oral explanation.

The experimenter will serve as a trained observer, making note of significant differences between the two groups. In particular, a tally will be kept of all student errors at the chalkboard, for comparison. Verbal classes will also be taped.

After each lesson, identical worksheets will be distributed to both groups for independent in-class practice with the material covered at the board. Students will be required to have each page checked and corrected before going on to the next. The experimenter will assist each teacher in the grading task.

Following instruction, the experimenter will administer a posttest of the same format as the practice sheets.
<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Move</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>↑↓↑↑</td>
</tr>
<tr>
<td>L</td>
<td>←↑</td>
</tr>
<tr>
<td>N</td>
<td>↑</td>
</tr>
<tr>
<td>R</td>
<td>↓←</td>
</tr>
<tr>
<td>Z</td>
<td>↘↑</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Move</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>↔↑</td>
</tr>
<tr>
<td>G</td>
<td>→↓</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Move</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>←↑</td>
</tr>
<tr>
<td>G</td>
<td>→↓</td>
</tr>
<tr>
<td>S</td>
<td>←↑</td>
</tr>
</tbody>
</table>
\[(1,6) \sim \eta (6,1)\]
\[(99,99) \sim \eta (99,99)\]
\[(100,101) \neq (101,103)\]
Lesson 11
Lesson #1
In this lesson, students learn to distinguish significant characteristics of geometric figures. The discrimination is made by sorting figures into classes according to important features. Following the teacher's example, students indicate which group a figure belongs with by drawing an arrow from the figure to its class. Figures are sorted accordingly as they are or are not closed, simple, curves, and figures which determine convex regions.

NEW TERMS: closed, simple, curve, convex, concave

VERBALIZE GENERALIZATIONS:

(1) A closed figure has the property that in drawing a picture of it, the chalk ends up at the same place it started.

(2) A simple figure is such that a picture of it can be drawn without ever tracing the same point more than once.

(3) A curve is a continuous figure, having no gaps, so that the chalk need not be lifted in drawing a picture of it.

(4) A figure which determines a convex region does not "cave in" whereas one which determines a concave region does.
Lesson #2
This lesson introduced the operations
\[
\uparrow \ \downarrow \ \rightarrow \ \leftarrow \ \uparrow \ \downarrow \ \rightarrow
\]
on a lattice of letters.

Using the argument as a reference point, the corresponding value in the range is the next letter in the direction specified by the arrow (\( A \uparrow = \_ \_ \_ \)). With just one example, students should catch on quickly to the nature of these operations from their symbols.

After practice with each operation singly, operations are then composed with themselves and with each other. Inverse operations are composed so that students will recognize that the same letter is simply returned to again (\( A \uparrow \downarrow = \_ \_ \_ \)). Other exercises are paired to suggest the commutative nature of these operations (\( A \rightarrow \uparrow \rightarrow \rightarrow \rightarrow A \rightarrow \_ \_ \_ \)). Finally, students learn to compose operations several ways, representing all possible paths between two fixed letters (\( A \rightarrow \uparrow = \_ \_ \_ ; A \_ \_ \_ = G ; \)
\( A \_ \_ = G \)).

NEW TERMS: operation, inverse, commutative

VISUALIZED GENERALIZATIONS:

(1) The nature of these operations is that using the argument as a reference point, the corresponding value in the range is the next letter in the direction specified by the arrow.

(2) When inverse operations are composed, the same letter is returned to again.

(3) These operations are commutative.

(4) Operations may be composed several ways to represent all possible paths between two fixed letters.
\begin{array}{cccccc}
F & O & R & S & T \\
K & L & M & N & O \\
P & Q & H & I & J \\
A & B & C & D & E \\
\end{array}

\begin{array}{c}
A \rightarrow = \\
\downarrow \downarrow \downarrow = \\
\leftarrow = \\
\downarrow \downarrow \downarrow = \\
\end{array}

\begin{array}{c}
H \downarrow \uparrow = \\
H \_ \_ \_ = H \\
H \_ \_ \_ = H \\
H \_ \_ \_ = H \\
\end{array}

\begin{array}{c}
N \rightarrow \leftarrow \leftarrow \leftarrow = \\
N \_ \_ \_ \_ \_ = N \\
N \_ \_ \_ \_ \_ = N \\
N \_ \_ \_ \_ \_ = N \\
\end{array}

\begin{array}{c}
H \rightarrow \rightarrow \leftarrow \leftarrow = \\
H \_ \_ \_ \_ \_ = H \\
H \_ \_ \_ \_ \_ = H \\
H \_ \_ \_ \_ \_ = H \\
\end{array}
The purpose of this lesson is to acquaint students with the ordered pair notation for naming points in the coordinate plane. Attention is restricted to points in the first quadrant with natural number components.

A lattice with one reference point identified is constructed by the teacher. He heavily chalks another point, reflects for a moment, then writes its ordered pair name directly below the point. \( (1,1) \) \( (2,1) \) Students follow suit, naming all points chalked in by the teacher. Ordered pairs are grouped so that it will be apparent that pairs with the same first component lie on the same vertical line, while those with a fixed second component are horizontally linear.

Having acquired the skill of naming a point given its location, the activity now switches to locating a point given its name. The teacher lists an ordered pair to the side, paused, then locates the point named by chalking it heavily. Having been cued to the procedure, students continue to locate and label other points. Ordered pairs are grouped to suggest that \( (a,b) \neq (b,a) \), except when \( a = b \).

On the worksheet, points on the same diagonal are grouped together so that the relationship between first and second components of the ordered pairs becomes apparent. To check for recognition of the pattern, points on the line beyond the portion of the lattice pictured must be identified also.
LATTICE: lattice, ordered pair, first component, second component, commutative

UNIFORMITY CONDITIONS:

(1) The relationship between a point of the lattice and its ordered pair representation is that the point in the m-th column and n-th row, with respect to the reference point (1,1), is named (m,n).

(2) Ordered pairs with the same first component lie on the same vertical line, while those with a fixed second component are horizontally linear.

(3) \((a,b) \neq (b,a)\) except when \(a = b\).
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\]
\[
\begin{array}{cccc}
(1,2) & (1,1) & (1,1) & (1,1) \\
(1,1) & (2,1) & (1,1) & (1,1) \\
(1,1) & (2,1) & (1,1) & (1,1) \\
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\end{array}
\]
Worksheet 13
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The image shows a diagram with coordinates indicating points on a grid. The points are labeled with coordinates such as (?, ?), (?, ?), (?, ?), (?, ?), (?, ?), (?, ?), and (?, ?).
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\[(1,2) \rightarrow (2,1) \rightarrow (1,1) \rightarrow (2,1) \]

\[
\begin{array}{c}
(2,1) = (1,2) \\
(1,2) \\
(3,5) \neq (5,3) \\
(5,3) \\
(6,4) \neq (4,6) \\
(6,4) \\
(66,78) = (78,66) \\
(89,49) \neq (49,89) \\
(999,1000) \neq (1000,999)
\end{array}
\]
This lesson introduces linear array operations using a machine format, where the input is the argument of the function, and the corresponding range value is the output. In order to give the required input or output, the student must discover the particular function which the teacher has in mind. After all the missing input and output entries have been filled in, the machine is labelled according to the operation it performs.

After practice with several different machines by themselves, machines are then composed so that the first machine's output is the second machine's input. Inverse machines are composed so that students quickly recognize that the resulting output is always the same as the initial input.

**NEW TERMS:** operation machine, input, output, composed machines, inverse machines

**VERBALIZED GENERALIZATIONS:**

1. The nature of these machines is that the output is obtained by performing the machine's operation on the input.

2. When inverse machines are composed, the resulting output is always the same as the initial input.
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X ↑↑ = ____
I ↓↓↑ = ____
G → → → ← ← ← = ____

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APPENDIX C

TRANSCRIBED TAPES OF VERBAL CLASSES
Lesson 51

Teacher 51
T: Okay. These figures I'm going to put in here are going to be called closed figures — they're called closed.

T: Whereas these figures over here will be not closed.

T: Okay, we'll be working with these. Okay.

T: Watch this. Around there and close it up.

T: Whereas over here, this figure is not closed.

T: This figure . . . around there and close it up.

T: This figure over here is not closed.

T: Start here . . .

T: Put here.

T: Now I'm going to draw a figure here, and I'm going to call on somebody to come up and draw an arrow to the closed figures or to the non-closed figures. Okay? Alright.

T: I'll do the first one. That'd be a closed figure, okay?

T: Okay. Alright, does anybody have the same answer? How many would have the other answer?

T: Good, that figure was closed.

T: Okay. Good.

T: Good, okay.

T: Is this too high for you?

T: Good.

T: Do you think you could do this without even seeing how it's drawn?

T: Do you see what we did for each?

T: Can you tell me something about whether a figure is — who can verbalize it?

T: Anybody have a question on what a closed figure is?

T: Who can — think they can tell us what a figure is that's closed?

T: Let's try. A closed figure is a figure that . . .

S: Is not open.

C: (Laughter)
T: Okay, but then we have to define what an open figure is and we say that an open figure is one that is not closed.

T: Let's see if you can try to figure if these ones are closed and these aren't.

T: Okay.

S: All the ends meet.

T: Okay, all the ends meet.

T: Okay, those that aren't closed, the ends . . .

S: Don't meet.

T: Don't meet. Okay, that was nice.

T: Somebody have another way they can describe a closed one?

T: That was nice.

T: Okay, let's — we can just save our clouds here.

T: We'll take up another concept. Let's begin with . . .

T: This time, we'll pick figures that are not simple — figures that are not simple — and some figures that are what?

C: Simple.

T: Boy!

T: Okay. Here's one that's not simple.

T: Here's one that's simple.

T: Here's another one that's not simple.

T: Here's one that's simple.

T: Here's another one that's not simple.

T: Here's one that is simple.

T: Okay, now we're going to draw some pictures and you find out what's going on.

T: Have you had one yet? It's your turn now.

T: Right.

T: Watch this.
T: Pardon me?
T: Robby.
T: Okay, here's another one.
T: Okay, Roy.
T: I'll let you do the next one.
T: Right.
T: Donald. You think you can get that right, Donald? Okay, we'll get back to you again then.
T: How many think Donald's right? How many think Donald's not right?
T: Want to fix that up, Donald?
T: Simple. Good, Donald. Now here's one right, Donald.
T: Serona.
T: How many think Serona's right? How many think she's wrong?
T: You got it right, Serona.
C: (Laughter)
T: Okay, I'm going to put this one over here, but that doesn't necessarily mean this — that one may be this.
T: Okay. If there's somebody who hasn't had one yet that didn't have their hand up. Okay.
T: Wayne, okay. Wayne can do this one. Wayne's going to try this one.
T: How many think Wayne's right? How many think — how many think that's something different than what Wayne has?
T: Okay, Wayne, you want to fix that up then? That's simple.
T: See if you can figure out now what these forms are.
T: Okay, two more.
T: Phillip, have you done one? Good, Phillip's going to do the first one.
T: Start doing the second one.
T: Right, that one's non-simple.
T: Celeda. Celeda's going to do this one.
T: Okay, Celeda got that right.
T: Now, who thinks they can . . .
T: Okay, Tim's going to tell us what he thinks simple figures are.
S: "Well the not simple ones are -- they cross, and the simple ones, they don't cross.
T: That's it. Alright, that's good. The ones that aren't simple, they cross. The ones that are simple cross.
C: Uh uh.
T: (Laughter)
T: You understand what I told you earlier, huh?
T: Okay, we'll keep our clouds there. At first I thought it was a balloon, but that seemed sort of a . . .
T: Okay. This time I'm going to . . .
T: What's that again?
S: What's the name of one?
T: Thank you. Well . . . Good, yeah. They decided to change this time. The words are at the end. Okay? After I get all these figures up here. Giving you a chance to puzzle out what they are.
T: Okay, these things are not curves.
T: These things are . . .
C: Curves.
T: Right on.
T: Okay, let's see how we do on something.
C: (Laughter)
T: Tim.
T: Okay, you got that one right.
T: Okay, Ruthie. Oh, excuse me.
T: Doug.
T: Good, Doug.
T: Okay, Teresina.
T: That's good. Teresina did that one.

T: Do you think you can do the next one?

T: Robby. Robby's up for a second one.

T: Pardon me? Have you volunteered? See if you can try to.

T: Good, Robby.

T: Let's see if some people will volunteer who didn't volunteer before. Did you do it before?

T: Stephanie.

T: Good, Delphany.

T: Cynthia. Okay.

T: Donna.

T: Good, Donna.

C: (Laughter)

T: Cindy.

T: Good, Cindy.

T: Phyllis, can you do this?

T: Dana.

T: Okay, good, Dana. That was a curve.

T: Okay now let's see who can tell us what a curve is.

T: Tim's pretty good at it — he gave us one before.

T: Let's... You want to try it? Phillip.

S: The not curves...

T: Good.

S: Are not connected. Curves are connected.

T: That's good.

T: You want to try something else Tim?

T: Okay, Donna.

S: Well, the curves, they have a curve in them somehow. Not curves,
they don't.

T: Well, it's more like Phillip said because some people wouldn't think of this part here as necessarily — of course, see some people wouldn't think of this as a curve in here.

T: So it's this idea of whether it's always connected, like Phillip said.

T: Well, if you think of connected as being curves, then you're right then.

T: Okay. Now worksheets.

T: No? Oh, I'm sorry, I ... We have one more to go here.

T: Do you want the name first, or do you want the name after?

C: First. After.

T: We'll put them both in there.

T: Those who would like to have the name after we fill in the clouds, raise your hand high.

T: Okay, the "afters" win already. (Laughter)

C: (Laughter)

T: These will be sort of interesting words, though. We can — nice to wait for these.

T: Okay. Here we go.

T: Okay these things over here are figures which are called concave figures.

T: Who can guess what these over here are called?

C: Not concave.

T: That's right, but we're going to give a special word for those figures which are non-concave. And those are called convex.

T: So those figures that are not convex are what?

C: Concave.

T: Those figures which are not concave are ... 

C: Convex.

T: Right on.
T: Here we go. See if you can do the first one.
T: Good.
T: Delphany.
T: Right, Delphany.
T: Serona.
T: Okay, Serona — that's real good.
T: Jim. Jim.
T: Good.
T: Jeffrey.
T: Good, Jeffrey.
T: Donald.
T: Good, Donald.
T: Christine.
T: Good, Christine.
T: Palmer.
T: Good, Palmer — that's right.
T: Let's put them right in the middle.
T: Coretta.
C: (Laughter)
T: Kaynetta. I'm sorry. No, that's alright — I can't read the signs.
T: Right, Kaynetta.
T: Okay, all those figures that go over to concave are concave. All those figures that are pointed for convex are convex.
T: Who can tell us what a concave figure is?
T: Okay, Christine.
S: Concave figures are figures that sort of — they sort of cave in on the side.
T: Hey, how 'bout that!
T: Did you hear what she said, Donald? Did you hear that?

T: She's telling us what concave figures are.

T: Did you hear what she said, Doug?

T: Doug, tell Donald and the class what she said about concave.

T: Okay, Michael.

S: Concave figures, they cave in.

T: Is that right, Donald? Do concave figures cave in? Here?

T: How about convex figures? Who can tell us what a convex figure is?

T: Eric.

S: They .

T: Pardon me?

T: Okay.

T: Somebody else.

S: Convex figures are the ones that aren't concave.

T: Okay, now we have some worksheets for you.
T: I have a secret. He have a secret that you can't talk.
T: You've got a secret? Oh.
S: You can't talk. You can't talk.
T: I can't?
C: ... C, E, E, ... H, I, J, K, ... P, Q, R, S, T.
T: I guess that's all we need today. Okay.
T: Now watch this one.
C: F.
T: Let's try this one now.
C: C.
T: Let's have somebody come to the board.
T: Palmer.
T: Watch — watch the example, then maybe ... You don't know, Palmer? Okay, well.
T: That's right.
T: Okay, Michael.
T: Let's see, that was right. Okay.
T: Let's look at some others now.
T: That's good.
T: Okay, Phyllis.
T: No, you watch when Phyllis is out there.
T: Okay.
T: Is that right?
S: Yes.
T: Jerrell, can you do that one?
S: Uh huh.
T: Okay. Okay.
T: Roy.
S: Give me one.
T: We have lots of them.
T: Good, Roy. Good.
T: Kaynette.
T: Celeda.
T: Donald.
T: Good. Good, Donald.
T: Christine.
T: Good, Christine.
T: Alright, these arrows are operations on those letters.
T: Who can tell us what those operations do to the letters? Raise your hand and tell about how those arrows work.
T: Donna.
S: One thing we know ... it points to the answer.
T: The thing it pointed to was the answer, okay.
T: Now let's erase all these.
T: We'll keep our — we'll keep our map on the board and we'll get some harder ones.
C: (Clapping)
T: Who came in?
T: Okay.
T: Let's see if you can get this one.
T: Jim hasn't had a turn yet. Raise your hand if you haven't had a turn yet.
T: Okay, we've got a few more problems for you.
T: Haven't you had a turn?
C: Yes. No. Yes.
T: Not today? Oh.
T: Good.
C: (Clapping)

T: Jim, Jim.

T: Delphany.

C: No.

T: Here we are. Stand over in front of it. It looks funny ... If you look straight — get right in front of it. Here we have A — over here's the arrow.

T: Good, Delphany.

T: Robby.

T: Good, Robby.

T: Raise your hand if you've had a turn.

S: If you what?

T: If you've had a turn.

T: Robby, you've had a turn? Okay.

T: Okay.

T: Roy.

T: Hey, I'm sorry.

T: Right.

C: (Clapping)

T: Doug.

T: Good, Doug.

T: How many had that? Keep trying.

T: Phillip.

T: Eric.

T: Okay, Eric, fix it up a little bit.

C: (Laughter)

T: That was good.

T: We call these things inverse ...
C: Inverse.
T: Operations.
C: Operations.
C: (Clapping)
T: This is the inverse of this one, and this is the inverse of this one, and this is the inverse of this one, and this is the inverse of that one.
T: Who can tell us something about inverse operations?
T: Okay, Christine.
S: Well, when you do it, after you figure out where the second arrow is, all you do is play like it was out of order.
T: Did you hear what she said, Donald?
T: Try this one now.
S: What is that?
T: That's a Z.
S: Oh.
S: What is that?
S: What is it?
S: A Z.
S: That's a Z with a line through it.
T: Okay, now here's the problem, not this.
S: Is that a Z up there?
C: (Laughter)
T: Dana.
T: Right, Dana. That's good.
T: Phyllis.
T: Can you do that?
T: Good, Phyllis. Right. Okay.
T: Who hasn't had one yet? Have you had one? Mike, have you had one
yet? have you had one?

T: Ruthie, will you do this one for us?

T: Kaynetta.

T: Okay, here.

T: Okay, Palmer.

T: Now watch this one.

S: Ruthie.

C: (Clapping)

T: Kaynetta.

T: Wayne.

T: See if you can figure it out . . . Look at the . . .

T: Right.

T: Richard.

T: Good, Richard.

T: Now who can tell us what they observed about these operations? In reference to examples like this?

T: Tim.

S: Well, the arrows are the same except they're in different places.

T: Good.

T: Then what happens to the result?

S: They go to the same place.

T: Right.

S: Give me one of those.

T: I think you all will get some on worksheets.

T: Okay, who can do this one?

T: Say, will you tell me your name?

S: Celeda.

T: Okay. Good, Celeda.
T: Okay.
T: Okay, that's good, but that one we've had already.
T: Okay, Ruthie.
T: That was difficult.
T: Good. Okay.
T: This is a challenger. Just one blank.
T: Roy.
T: Try to have them ready at your seat so then when we call on you, you come up and so...
Lesson 12

Teacher 12
T: Now this is going to be a special operation.

T: And to get the answer to this — the answer to this is F, okay? Alright.

T: Okay, who can tell me the answer?

T: Okay, Ferman.

T: Right.

T: Okay, that's the right answer. Very good.

T: Alright, how would you describe this operation?

S: It would be like A and then the number — letter on top of it, then that'd be it.

T: That's good. That's correct, okay.

T: Okay, now we are going to take a different operation.

T: Alright, Sandra is going to do that.

T: Right.

T: Now we'll do one more of this type.

T: Okay.

T: That's good. That's the right one.

T: Now a different operation.

T: Let's see, how would you describe this operation here?

S: One down below it.

T: One down below it — that's right. One down below it — that's correct.

T: Okay, now we are going to take an operation like that.

T: Okay, Joey.

T: That's right. Thank you. Correct.

T: More like that? That's a different operation.

T: How would you describe this operation, like that? What would you say?

S: The next one beside it.
T: That's the one you see — the letter beside it. Okay.

T: To the left or to the right?

S: To the right.

T: To the right — you are right — to the right. If you say to the right, you are right.

T: Okay, this one here.

T: Alright, Cassandra. Barefoot Cassandra, eh?

T: That's right, now — very good. Okay.

T: Now we are going to take . . .

T: How would you describe an operation like the last one over there?

T: Paula, how would you describe that?

S: The arrow is pointing to the left of . . .

T: Very good — to the left. Alright.

T: Now a different group. Like that.

T: Okay, Steve.

T: Steve is right. That's good.

T: How would you describe an operation like that?

S: Going up like that would be the answer.

T: Up like that? I know.

S: Up and slanted.

T: Slanted, slanted, huh? Okay, that's good.

T: Here is a different one.

T: Alright, Steve.


T: Now we are going to do like that.

T: Okay. Alright, Teresa.

T: Right, L. That's good.
T: I is right — that's the one.

T: How would you describe this operation here? Going this way?

T: Russell.

S: Well the answer is diagonally down on the right.

T: Diagonally down on the right — that's a good description. Okay.

T: Now we're going to do two operations in a row.

T: Okay. I'm going to tell you that they are going to go like this — they are going to take A and go ... then I know what ...

T: Alright, okay.

T: C is right — good.

T: Q, right.

T: Okay, Laurie.

T: That's the letter T, huh? I'll buy that — that's good.

T: Now. How about that?

T: Beth, okay.

T: D is right — very good. That's good.

T: Now, who are we going to pick?

T: Alright, Tom.

T: No, erase that.

S: That was another one.

T: Pardon me?

S: That was another one.

T: Well, well ... Read it — look at it again.

T: See there are two operations — we go once and we go twice.

T: And I disagree with your answer. There are several — there are some friends here of yours who disagree with you too. I don't know, maybe they aren't your friends. I don't know, but they disagree.

T: Alright.

T: Okay, okay — somebody else. Okay, Joel — you do it, okay?
T: Right — that's the way. That's good. Okay.
T: Donna.
T: Right, P. Alright, that's the answer.
T: Russell.
S: Is that a W?
T: That's a W.
T: Right, you got the right answer even though there is no W up there.
T: This is Y. And don't ask me why this is Y.
C: (Laughter)
T: And Y is the answer — good. And I won't ask you why Y is the answer, okay?
C: (Laughter)
T: Now watch carefully. I am going to do this whole thing. One, two, and Z — do you agree?
C: Yes.
T: Okay. Now, I'm going to start with Z again, and I'm going to finish with Z, and I want you to fill in two operations different from the ones I have already. Alright?
T: Okay, Jody.
S: Do you want then different?
T: Yes, different from the ones ...
T: That's good. Oh, that's great.
T: Still different.
T: Okay. Still different from the two.
T: Good, alright.
T: How about still different?
T: Good.
T: Can somebody tell me what these two operations do? They kind of do one thing all the time.
T: Beth, what do they do?
S: They're just the opposite.

T: They're just opposite, yes. They just come back to the same letter they started from. Yes, they are opposite.

T: And we call them — there's another word for opposite — we call them, like that.

C: Inverse.

T: Inverse, yes. They are inverse operations — like down and up — inverse operations. One does something, the other cancels it, right? Inverse operations. Okay.

T: Okay, Joey.

T: G, right. Okay.

T: Now that.

T: Good.

T: So I'm going to do like this — I'm going to write this and this. Here, I'm going to write this, and then I'm going to say equals and I'm going to say not equals, and which is right? Not equal or equal?

C: Equal.

T: You are right. Okay.

T: Another one like that. I want the answer to that.

T: Okay, Laura.

T: L is right. That's good.

T: Now I'm going to say this.

T: Alright, Tony.

T: I don't like it. Take a look at it again and see whether you can improve on it.

T: Sorry, sorry — you are right. But I'm going to change the problem, okay? Alright. I'm going to change that to this.

T: Take a look — take a look at how it starts.

T: L is right. That's the right answer — that's the one. Okay.

T: Now we are going to write this and this, and then I write this, and I want somebody to circle the right thing.
T: Okay, Robyn, you are going to circle the right thing for us, okay?
T: That's right. Okay.
T: So we ... like that. Up here will be ...
T: Circle in the right place.
T: Okay. Circle in the right place.
T: Good, that's the one.
T: Okay, we're going to take Z, this way, then this way. And Z this way and this way.
T: Okay, Deborah. Ready to circle?
T: Right, that's good. That's good.
T: W going this way, right? Then W going this way and this way.
T: Then I need a circle.
T: Okay, a circle — a circle for Ferman. My circle.
T: I kind of like a circle — I kind of like a circle in that place. I don't like the circle in that place.
T: Okay, Lola, you want to correct that?
T: Watch, Ferman — see what Lola will do.
T: That's where the circle goes. Same thing. You got the same answer on each side.
T: Okay, was somebody watching how we were doing these over here and tell me what you saw in these? How these operations were arranged? Can you tell me how it —— what we did there?
T: Beth.
S: The inverse.
T: The what?
S: Inverse.
T: Well, I'm not sure about that. We did something with these two operations. We did something ... 
T: Russell.
S: If they land on the same thing, we put equal.
T: That's correct. But, that's right. We put equal because we hoped it will land on the same thing, Russell says.

T: But if you look at these two, they are arranged in a kind of a special way. Could you tell us what special way?

T: Alright, Sandra.

S: They're in a different order.

T: They're what?

S: They're in a different order.

T: Right, they're in a different order. I reversed the order. Here I had this first, then this, and here, the other way around — this first, then this. See, that's the way.

T: We have a special name for operations like that which you can switch the order to get ... They are ... There's a special funny name. It's called — the operations are called ...

C: Commutative.


C: Commutative.

T: That's good. Yeah, that's right. Okay, that's what we say — commutative operations.

T: It means it doesn't matter in which order you do them, you get the same answer. Cause we were writing equal and I said "right" and I think I know the answers. What do you think? Well, it's not my place to ...

T: Okay, we are going to do another one. Start with L and go two operations and we get R. And I want somebody to fill in two operations for us. And they are going to take us from L to R.

T: Donna wants to do it for us, okay?

T: That's good — that will take us from L to R.

T: How I want a different way to get from L to R. And everybody wants to get out of L, when I say L. Right? We want to go from L to R.

T: Okay, Teresa.

T: That's good — that's another way of doing it.

T: Now, one operation to go from L to R — just one.
T: Okay, Paula.
T: That's good. Good.
T: Okay, now I'll start with S (right?) and I want to end up at I.
T: That's good — that's a good way to go from S to I.
T: Now I want to go a different way from S to I with two operations.
T: Okay, Steve.
T: That's good.
T: And still a different way to go from S to I.
T: Okay, Joey, since you're already out of your seat.
T: That's good — that's another way to . . .
T: So there are different ways of getting from one letter to another, are there not? You can go different ways.
T: Okay, let's try some more like that.
T: I start with N, and I use two operations, and I finish at B.
C: (Laughter)
T: Good — that will take us to B, I agree.
T: Another way from N to B, a different way.
T: Okay, Richard, ready?
T: Good, that will take us to B also.
T: Now watch — N, one, two, three.
T: Okay.
T: Ah, that's good — that will take us to B.
T: And again from N to B a different way.
T: Okay, Laura.
T: That's good — that should take us to B. Let me just check to be sure. Let's see, there . . . Right. Laura is right.
T: And one more different way from N to B. From N to B, or to B or not to B, that's the question first, okay? Three ways to do it.
T: Let's see — would you check that first before you run away? N . . .
T: That's good. That's correct. Okay.

T: And now we are going to find out what you did, right? We are going to give you a little something to work on. Right?
Lesson 12

Teacher #1
T: Today we're going to draw tiny, okay?
C: Tiny?
T: This thing across here, who knows the name of them?
S: The name of them?
T: That collection of dots --- that would be a row.
C: Row.
T: Row. Going across --- that's a row.
T: Here's another row.
T: Now we go all the way down --- what would we call that?
C: Column.
T: Good. Boy!
T: Starting over here's a column, here's this column, this column, column, the last column, and the whole works is a lattice --- the dots altogether, they make a lattice.
T: Okay? Now we're ready to go.
T: Can you see that, Phyllis, from where you are? That's good.
T: We can't write much larger than that today.
T: That's good, the ordered pair (1,1).
S: Dr. Wills, you left some papers here yesterday.
T: Still there?
S: They were on the table.
T: Oh, thank you.
S: You're welcome.
T: Here's one for somebody in the class to do.
T: You have the right answer?
T: Fill in the empty space there.
T: Doug.
T: Right, Doug. Good, Doug.
S: That's simple.

T: Here's a harder one.

S: Uh oh.

T: Willie.


T: Okay, we'll try to make it a little more harder this time.

T: Donna.

T: Good, Donna.

T: Teresina.

S: I've got to think about it.

T: Okay.

T: Jeffrey.

S: Jeffrey?

T: Yeah, would you like to have a different one? Okay.

T: That's right.

T: I want somebody who can really do this. A right answer.

T: Wayne. Good.

T: Good. Okay.

T: Eric.

T: Good, Eric.

T: Kaynetta.

T: You had it right at first. Good.

T: Robby.

T: Good, Robby.


T: No, no. Fix that up.

T: Good.
T: Tim.
T: That's good, Tim.
T: Cindy.
T: Good, Cindy.
T: Now we go way over here.
T: Dana, do you think you can handle it?
S: Yes.
T: Okay.
T: Good, Dana.
T: Doug.
T: Good, Doug.
T: Okay. Who can tell us something a little bit about when we have the second component here?
S: What?
T: Okay, Christine.
S: Well, like if you start — if you start on the bottom row, you know that that's the first row, so the second number would always be number one and the second row, it'd always be number two.
T: The second component, right? Okay, if they have the same second component.
T: This is 3, 3, 3, 3. What's true about the ordered pairs?
S: With what row?
T: With the same second component.
T: Okay, what about if the second component is 2? What do we know about the pairs of numbers whose second component is 2?
T: Robby.
S: Each column of rows has a number, like 3 there.
T: Okay, who can tell us what they know about all the ordered pairs of numbers that have a second component 2?
T: Kaynetta.
T: Phillip.
S: Well like if you go up the column you've got . . .
T: You're talking about something else. Let's look at those ordered pairs that have second components 2, and tell me something about those ordered pairs — the dots that correspond to those ordered pairs.
T: Cindya.
S: That other time one dot, if you put a one in front and then you put the row, then you would know . . .
T: That didn't answer my question though.
T: Robby.
S: The first row has number one for its second component, and the first . . .
T: Okay, that's right.
T: Let me tell you something about those numbers that are . . . For this dot — it corresponds to the ordered pair (3, 2) and 2 is called the second component — second component.
S: Component.
T: What's the second component of the ordered pair that corresponds to this dot?
C: 2.
T: Okay. What do you suppose you'd call 3?
C: The first component.
T: And also, when you have a pair of numbers like this — where you have a first component and a second component — that's called an ordered pair.
S: An ordered what?
T: An ordered pair of numbers — an ordered pair. Okay.
T: Let's see if someone can do this one for me. If they're . . .
T: Phyllis.
T: Fix it up, Phyllis.
T: Good.
T: Ruthie.

T: Do you understand this now, Teresina?

S: The times you asked me see, I couldn't see the numbers. You were in the way.

T: Oh, I'm sorry about that. I didn't even know that.

T: Good, Ruthie. That's beautiful.

T: Teresina.

T: No, that's wrong.

T: Mike.

T: If you raise your hand, that means you can do something. You don't ask me what the question is after, after.

S: What's he doing?

T: He had his hand up — I guess he thought there was a question.

S: Oh.

C: (Laughter)

T: Okay, let's get the questions first, then we can volunteer.

T: Willie.

T: Good, Willie.


S: Jerrell.

T: Jerrell.

T: That's right, Jerrell, so far.

T: Okay, Jerrell, you can stand on this chair.

T: Take your time.

T: Good.

T: Okay. Now who can tell us something about the dots that correspond to the ordered pairs with the same first component?

S: What?

T: Who can tell us something about the dots that correspond to the
ordered pairs with the same first component?

T: Here's a dot that corresponds to the ordered pair with 2, first component 2, first component 2, first component 2, and first component 2. What do we notice about these dots?

T: Phil.

S: The second dot in, well whatever row its in, it's ...

T: It's ...

T: They also all lie in the same what?

C: Column.

T: Column, right.

T: And here's all the dots with the first component what?

S: Dot.

T: What is the first component of all these?

C: One.

T: And they all lie in the same ...

C: Column.

T: Column. Okay.

T: Christine.

T: Good, Christine.

T: Okay, Richard.

T: Phyllis.

T: You do this one for us.

T: Mike.

T: You watch Mike now, Jerrell.

T: Right.

T: Now ... Work it out.

T: Good, Tim.

T: Okay, here we go. Okay.
T: Dana.

T: Good job, Dana. That's very good. Okay.

T: Let's try the reverse of this. Let's try the reverse of that. That would be 6, what?

C: 5.

T: Celeda.

T: That's good, Celeda.

T: Let's first start naming the dot, then find — first find the dot. Let's just find the dot then . . .

T: Okay now let's see if you can do this one. Compare them — are they equal or are they not equal?

T: Jeff. Okay, will you do that one then?

T: Jeff, is that the same dot? Is this dot the same as this dot?

T: Okay. There you go, good.

T: Cindy.

T: Don't raise your hand until you can really go up there and dash it off.

T: You people are writing way too small today, so you need writing that doesn't interfere with the others, that's good.

T: Okay, what should we try now?

C: (7, 3)

T: (7, 3) is the reverse of (3, 7). Let's put that down.

S: Reverse.

T: The reverse.

T: Phillip.

T: Can you do that one for us, Kaynetta?

T: Good, you got it on the right dot.

T: Good.

T: Alright, now we're going to do equal or not equal.

T: We want somebody who can come up to the board and do it right away.
T: Kaynette.
T: Good. Someone else.
T: Right.
T: She came right up there and put it right down.
T: Okay, another one. What should be this one hero?
C: (77,88).
T: Right on.
T: Jerrell.
T: Right.
T: This one here?
C: (99,100).
T: Let's see, who hasn't had one? Who did good that hasn't had one? Palmer, you had one before. Who hasn't?
T: Roy hasn't had one. Have you had one? Let Roy do this.
T: Good, Roy.
T: Now let's not try to exercise our vocal chords. Let's try to think of what we've seen here.
T: What can we say then about when we reverse the components of an ordered pair? Do we get the same dot or do we get a different dot?
C: Different dot.
T: Different dot.
T: Can you think of a case where if we reverse the components, we get the same dot?
T: Donna.
S: (8,8)
T: Good. Hey, what about that!
T: Okay, (8,8). And over here, what would we have?
C: (8,8).
T: (8,8), and let's put equal and not equal.
C: Equal.
T: Right.
Lesson 13

Teacher 22
T: Okay. (1,1) goes with this dot.
T: (2,1) goes with this.
T: Now I want to know what goes with that dot.
T: Okay.
T: (3,1), that's right.
T: Okay. Now do this.
T: (4,1), right.
T: You know what we call a bunch of points like that? There is a word for it. We call it this . . .
C: Lattice.
T: Lattice, yes. The lattice of points, right.
T: And each one of those pairs of numbers, we call an ordered pair. Why do you think we say ordered?
S: It's in order.
T: Yes, it's in order. There is a first number; there is a second.
T: Like over here — what is the first number in this one?
S: 4.
T: 4, and the second number?
S: 1.
T: Right, yes.
T: Okay. Do this one.
T: Take a look at it, take a look at it. Don't rush away yet. Come back, Steve. Let's see. It's not that — it's something else. Take a look at it.
T: Okay.
T: (6,1), that's right.
T: See that Steve?
T: That's correct. Okay.
T: This one.
T: \((8,1)\), good.
T: This first number we call the first \ldots
C: Component.
T: Component — first component.
T: And this we call \ldots
C: The second component.
T: First component, second component — in an ordered pair. Alright.
T: Okay, Donna.
T: Right, \((1,2)\). That's right.
T: Tom. No? Try again.
T: You're not sure, huh?
T: Can somebody else do it?
T: Deborah, okay. See whether you can do it.
T: \((2,2)\) is right. \((2,2)\) is right. That's what we want — that's right.
T: Okay.
T: \((3,2)\) is right. Very good.
T: Okay, Laura.
T: Very good. \(7\) — that's right.
T: Okay, Joey.
T: \((1,3)\), right. \((1,3)\), right.
T: Tom.
T: \((3,3)\), right.
T: Paula.
T: \((8,3)\), right.
T: Okay. See the first number always tells the vertical thing, right? And the second number right here tells the horizontal.
T: Sometimes you say this is the column and this is the row. That's the way we tell, don't we?
T: Alright, we'll do a few more. We want this one over here.
T: Steve.
T: (1,4), right.
T: (1,5), right.
T: (1,7), right.
T: What can you tell me about the first component of all the ordered pairs in the first column? What about it?
T: Jinx, yes.
S: The same number.
T: They're all the same number.
T: What's that number in the first column?
S: 1.
T: 1.
T: What's that number in the eighth column?
S: 8.
T: 8, right.
T: That's the way it goes. Okay, that's good.
T: Good, (2,5). That's correct.
T: Okay, Tamara.
T: See whether you can reach up there, okay? Think you can do it?
C: (Laughter)
T: Did I hurt you? I'm sorry. I shouldn't walk backwards like that. Are you okay?
T: Okay, that's right.
C: (Laughter)
T: Okay, so we see that all these points in this — like first of all they have the same which component?
C: One.
T: Second component is the same.
T: Is the second component all the same here?
C: No.
T: Is the second component?
C: Yes.
T: What is it?
C: 3.
T: 3.
T: What's the first component in here?
C: 3.
T: What's the first component in all of these?
C: 3.
T: Right. Very good.
C: (Laughter)
T: Good. (3, 7), that's right.
C: (Laughter)
T: I'm going to do this one. Let's see, we are going to find where it goes.
T: Okay. Now I'm going to have you do one.
T: Okay, Jody.
T: We want (5, 6) — I want you to find (5, 6) for me, okay?
T: Well . . .
T: Would someone like to correct?
T: That's good — that's where (5, 6) goes. That's good.
T: Okay. Now (6, 5).
T: Alright, Teresa.
T: (6, 5).
T: (6, 5), right. That's good — that's where (6, 5) goes. Thank you. That's good.
T: That's (6,5) and this is (5,6), okay?
T: Now, (7,8).
T: That's good.
C: (Laughter)
T: No — (7,8).
T: Okay, let's try somebody else.
T: Okay.
T: Good — that's (7,8), right.
T: And one more.
T: Let's see. Let's see, Laura.
T: Right, that's the one. Very good, (6,7). Good. That's (8,7).
T: And (7,8).
T: Okay.
T: (5,6), right. Good.
T: Good.
T: Good, that's right.
T: Not equal, huh? That's good.
T: So we see when we — when we switch around the components then what we get, we get not equal ordered pairs, right?
C: Right.
T: Okay. Thank you.
Lesson 24

Teacher 12
T: Okay, I'm going to put 1 into the machine and 2 comes out.
T: I'm going to put 2 into the machine...
C: And 3 comes out.
T: And 3 comes out.
T: I'm going to put 3 into the machine and what comes out?
T: 4, right. That's good.
T: I'm going to put 5 into the machine and Lela comes out.
C: (laughter)
T: 6 is right.
T: I'm going to put 10 into the machine and Joel comes out.
T: Alright. Okay.
T: I'm going to put 30 into the machine and Russell is going to come out.
T: That's good.
T: Aha, 71 comes out of the machine and what goes in?
T: Laura goes in.
T: 80 is right. That's good. 80 is right.
T: 91 comes out and what went in?
T: Roderick went in.
T: 90, right. 90 is right — very good.
T: I'm going to put in 100 and what comes out?
T: Ferman is going to come out.
T: 101 is right. Okay.
T: We're going to call this machine add one machine — an add one machine — that's the name of the machine.
T: What does the machine do?
C: Add one.
T: Add one. Whatever you put in, it adds one. Okay, good.
T: Now I'm going to take 4, put it into the machine, and 1 comes out.
T: I'm going to take 5, I'm going to put it in, and 2 comes out.
T: Now I'm going to take 6, and I'm going to put it in, and . . .
C: 3 comes out.
T: 3 comes out.
T: I'm going to put 7, and what comes out?
T: Tamara comes out.
C: (Laughter)
T: 4 is right. Thank you, madame.
C: (Laughter)
T: 10 is going to go in and Joey is going to come out.
T: 7 is right.
T: I'm going to put 33 into the machine and what comes out?
T: Lee comes out.
T: Do you want to come? No?
C: (Laughter)
T: 30 is right. Very good.
T: Aha, 50 came out — what went in?
T: Steve went in.
T: 53 is right.
T: 93 went in — what came out?
T: Teresa — for the president of the United States.
C: (Laughter)
T: 90 is right. Right.
T: 100 came out — what went in?
T: Jinx went in.
T: Okay. Now somebody is going to tell me right here what kind of machine this is.
T: Seth is going to tell me. What does the machine do?

T: Subtract 3 is right. The machine subtracts 3. Whatever it gets, it subtracts 3. Okay.

T: Just change the machine.

T: I'm going to put 1 in and 2 comes out.

T: I'm going to put 2 in and 4 comes out.

T: I'm going to put 3 in and 6 comes out.

T: I'm going to put 4 in and what comes out?

T: Tom.

T: 7 does not come out — something else does.

T: Okay, Lela, see what you can do.

T: I would say that's right. That's what comes out — 8.

T: I'm going to put in 5 and what comes out?

T: Robyn comes out. Okay, here we go.

T: 10 is right.

T: 1/2 comes out — what went in?

T: Russell went in.

T: 7 is right.

T: 6 went in — what came out?

T: Tony is going to come out.

T: 12 is right.

T: 18 came out — what went in?

T: Jody went in.

T: 9 is right.

T: And 10 went in, and what came out?

T: Paula came out.

T: 20 is right.

T: Now what machine is that?
T: Sandra.
C: (Laughter)
T: And, multiply by 2 machine is right. Thank you, Sandra. That's a multiply by 2 machine. Okay?
T: Alright, now we're going to do — you know what we're going to do? Nobody knows what we're going to do. Who's going to tell them?
T: We are going to hook up two machines.
T: Here is one machine — now that's a beautiful picture of a machine.
C: (Laughter)
T: But that's going to be what kind of machine?
C: Plus 1.
T: Then we're going to put another machine in here — like that.
T: Then this machine is going to be a plus 1 machine.
T: And I'm going to put 1 into the machine, in, and out, and goes into this machine — like that, and comes out here.
T: Now I want to know what comes out of the first machine and then what comes out of the second machine.
T: Who is wise here? You are wise, of course. Whoooooo . . .
C: (Laughter)
T: Right, 2, 3. That's right.
T: I'm going to put 2 in here.
T: 3 and 4, right. That's right.
T: 5.
T: 6 and 7 is right.
T: Out of the second machine came 11 — what went in and in?
T: Mr. Jody is going to . . .
T: Aha, 31 came out of the first machine — what went in here?
T: Mr. Tony.
T: 30 went into the first — absolutely right.
T: 50 went into the first machine — what came out of the second machine?
T: 52 is right — 52.
T: 60 came out of the second machine ...
T: Teresa is going to come out of the second machine.
T: 62 is right.
T: And out of the second machine came out 82 — what went into the first machine?
T: 81? Aha!
T: 80 is right. That's good.
T: And 102 came out of the second machine — what went into the first machine?
S: Dr. Nichols ...
T: No, Dr. Nichols didn't go into the first machine.
T: 100 went into the first machine. Okay, that's good. That's good.
T: Now we are going to fix the machines differently. Different machines.
T: Okay. The first machine is going to be add 3, and the second machine is going to be ...
T: And we're going to put 1 into the first machine ...
T: Aha, aha. And that is not the case.
T: 1 goes into the first machine and the second machine is add 3, so ...
S: 1 and 2.
T: No. No.
T: What does the machine do? The first machine?
T: Add 3. 4, right.
T: What does the second machine do? Take away 3, okay. 1 is right. Okay.
T: Alright, 2 goes into the first machine — what comes out?
T: 5, right.
T: 9 goes into the first machine — what comes out of the second machine?

T: 9 is right.

T: 33 goes into the first machine — what went into the first machine?

T: 33 is right.

T: 67 goes into the first machine — what comes out of the second machine?

T: 67 is right.

T: 81 came out of the second machine — what went into the first machine?

T: Lee is going into the first machine.

T: 81 is right.

T: And 99 went into the first machine — what came out of the second machine?

T: 99 went into the first machine — what came out of the end?

C: (Laughter)

T: 99 is right.

T: Okay, now we're going to find out what you will be doing.
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VITA

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